TRANSLATION BETWEEN BASES

Base 5 to Base 10

To convert a number from base 5 to base 10, multiply each digit by the place value it represents.

Consider **14302 (base 5)**.

1	**4**	**3**	**0**	**2**
5^4	5^3	6^2	5^1	5^0
625	125	25	5	1
Stack of Cubes	Cubes	Flats	Longs	Units

So:

$$\mathbf{1 \times 625 + 4 \times 125 + 3 \times 25 + 0 \times 5 + 2 = 102}$$

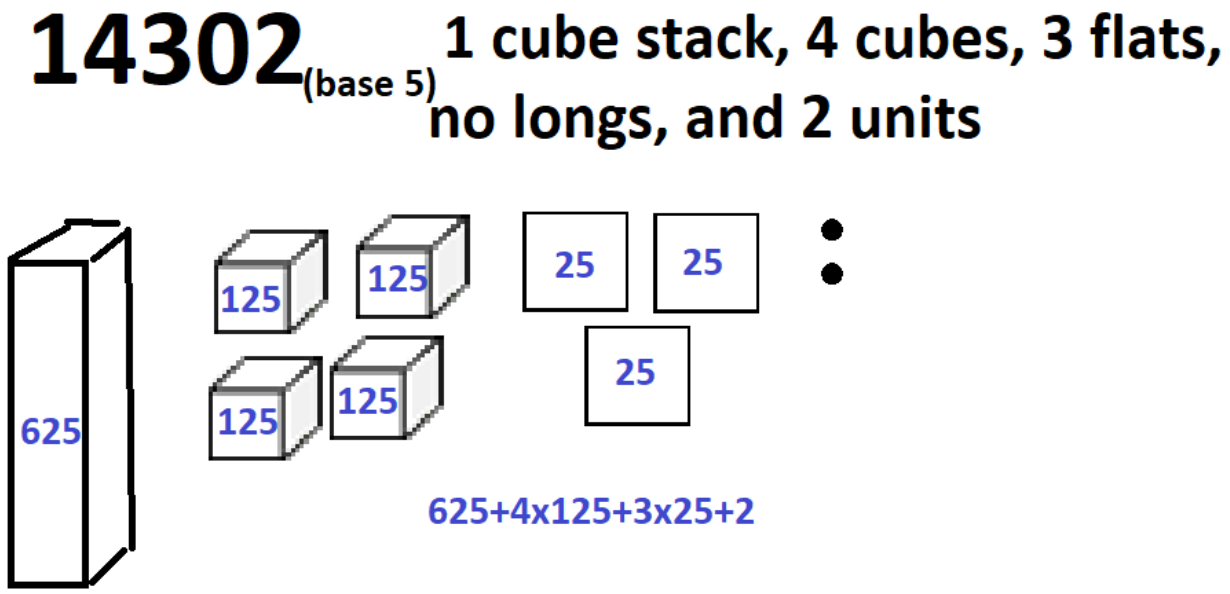

Figure 1:14302 (base 5) conversion to base 10

Base 10 to Base 5

To convert a number from base 10 to base 5, there are a couple of options. Let's do the same number both ways.

Consider **347.** Note that since there's a 7 in the number, it's not already a base 5 number.

Option 1: Illustrates the Concept

We want to represent our number with the *fewest* number of blocks. So we start with the biggest block that will fit, and subtract it from our number until no more fit. Then we move to the next size down, etc.

From smallest up, our base 5 blocks stand for 1, 5, 25, 125, 625, etc. 625 is too big.

So how many copies of 125 (cube) fit in 347?

$$347 - 125 = 222$$
$$222 - 125 = 97$$

97 is smaller than 125, so no more fit. So we have 2 cubes.

The next size down is 25 (flat). How many copies of 25 fit into 97 (what's left)?

$$97 - 25 = 72$$
$$72 - 25 = 47$$
$$47 - 25 = 22$$

22 is smaller than 25, so no more fit. So we have 3 flats.

The next size down is 5 (long). How many copies of 5 fit into 22?

$$22 - 5 = 17$$
$$17 - 5 + 12$$
$$12 - 5 = 7$$
$$7 - 5 = 2$$

2 is smaller than 5, so no more fit. So we have 4 longs. The remaining 2 is converted into units.

So 347 (base 10) looks like 2342 (base 5).

This method works from the largest block down.

Option 2: Quick Calculation

This method is the quickest way to get your answer, but it's harder to see why it works.

Divide your number 5 by 5, recording the remainder each time. Then divide the answer by 5, etc. until you're down to a number less than 5.

$$347 \div 5 = 69\ R\ 2$$
$$69 \div 5 = 13\ R\ 4$$
$$13 \div 5 = 2\ R\ 3$$
$$2$$

Read your answer from the bottom up, using the remainders.

So 347 (base 10) looks like 2342 (base 5).

Note: You can check your work by converting the base 5 number backwards.

This method works from the smallest block up. When you divide by 5, you're essentially regrouping to the next size block.

Any Base to Base 10

To convert a number from any base to base 10, multiply each digit by the place value it represents. The only difference is adjusting for place value

Consider **3A76 (base 12)**. (Remember that A stands for 10.)

3	**A**	**7**	**6**
12^3	12^2	12^1	12^0
1728	144	12	1
Cubes	Flats	Longs	Units

$$3 \times 1728 + 10 \times 144 + 7 \times 12 + 6 = 6637$$

So 3A76 (base 12) looks like 6637 (base 10).

Base 10 to Any Base

Either of the 2 options discussed above will work here, too. Instead of working with 5, however, you're working with whichever base you're converting to.

Consider **305.** Let's convert it to base 8.

In base 8, we're looking at 1 (unit), 8 (long), 64 (flat), 512 (cube), etc. (multiply by 8 going up). So, since 512 is to big, start at 65:

$$305 - 64 = 241$$
$$241 - 64 = 177$$
$$177 - 64 = 113$$
$$113 - 64 = 49$$

So 4 flats. Now longs:

$$49 - 8 = 41$$
$$41 - 8 = 33$$
$$33 - 8 = 25$$
$$25 - 8 = 17$$
$$17 - 8 = 9$$
$$9 - 8 = 1$$

So 6 longs and 1 unit.

So 305 (base 10) looks like 461 (base 8).

Any Base to Any Other Base

I recommend converting from the first base to base 10, and then from base 10 to the second base.