TRANSFORMATIONS

Transformations are a topic I didn't encounter until late in college. However, in my first year teaching, I had to teach them to eighth graders—and they already knew the concept, they were just learning the formal terminology. What you learned in school isn't necessarily what you're going to be teaching. Be prepared to be flexible and willing to try new things.

Transformations are ways of moving shapes, usually without changing their size. You probably already know them: translations (slides), rotations (turns), and reflections (flips); sometimes scaling is includes (adjusting size in proportion). For the three main transformation, the size of the shape is maintained. Translations retain the orientation as well.

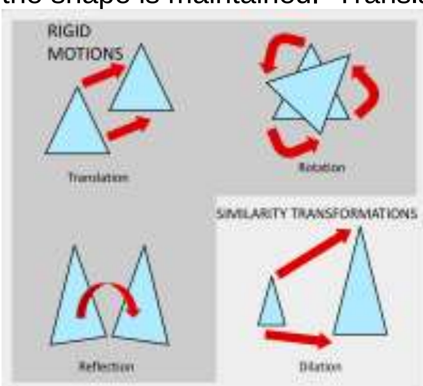


Figure 1: Transformations
https://commons.wikimedia.org/wiki/File:Similarity_and_congruence_transformations.svg
CC0 1.0

Translations

A **translation** can be described as a sliding motion. Each point is moved the same distance and the same direction. In Figure 2, note how each point of the triangle F moves 3 points right and 4 points up to its image F'.

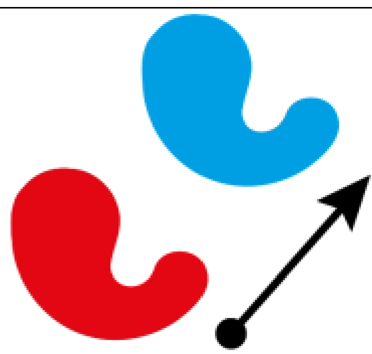


Figure 2: Translation
https://commons.wikimedia.org/w/index.php?curid=35017753
CC BY-SA 4.0

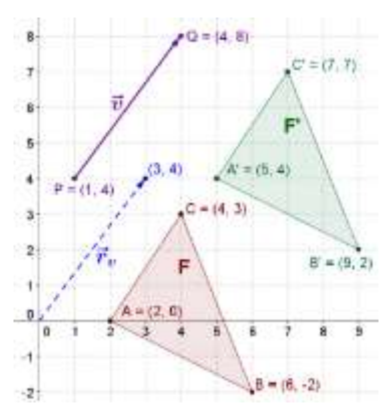


Figure 3: Translation on graph
https://commons.wikimedia.org/wiki/File:Translation_geometry2.png CC BY-SA 3.0

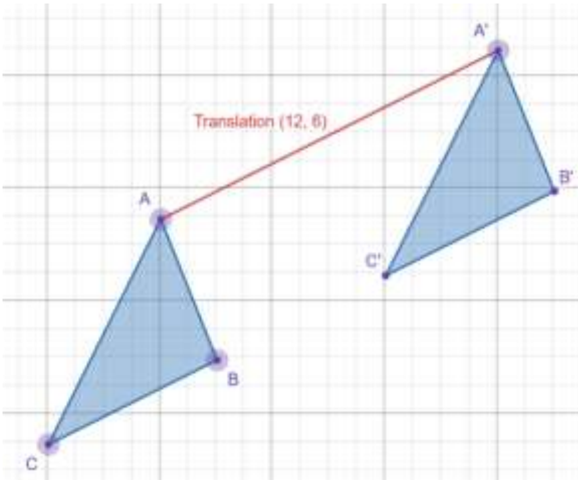


Figure 4: Translation (12, 6)

In figure 4, the triangle is translated 12 units right and 6 up. You can physically count the distance from the original point to its image.

Translations can occur with space figures as well as plane figures. Just like in the case of 2-D figures, the translation is described as a sliding motion of points in space in the same direction for the same distance. Visualize a box on a conveyor belt, or a child going down a slide.

Reflections

A **reflection** about a line is mapping that can be described as folding. If the pane if folded around the line of reflection, each point will align with its image. Any point of the reflected on the line of reflection is a fixed point; it doesn't change. If you draw a line from a point in the figure to the line of reflection that is perpendicular to the line of reflection, the image of the point is the same distance from the line of reflection along the perpendicular.

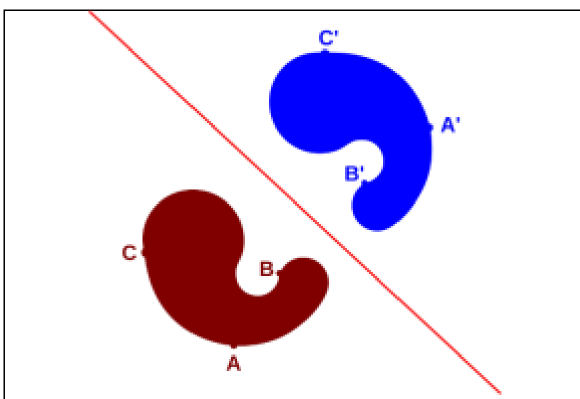


Figure 5: Reflection
https://commons.wikimedia.org/w/index.php?curid=139044052

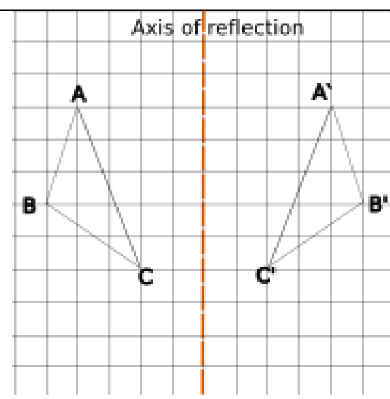


Figure 6: Reflection on a graph
https://commons.wikimedia.org/wiki/File:Reflectional_transformation.svg

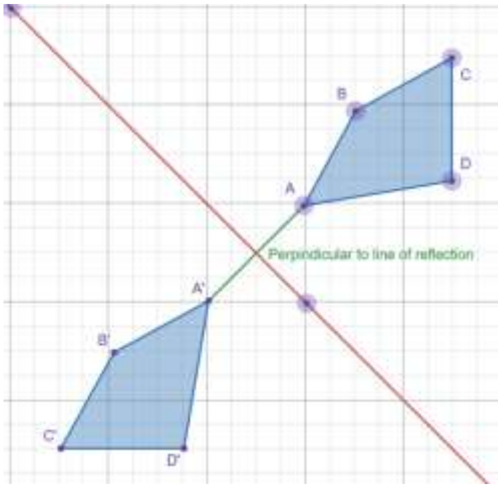


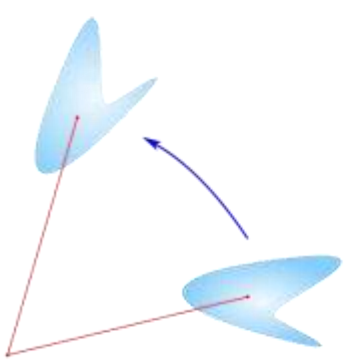
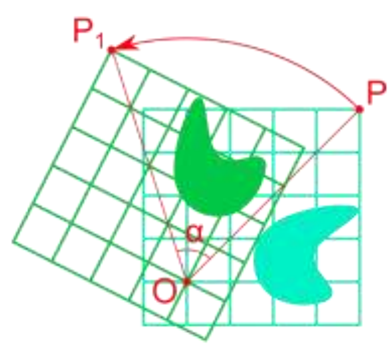
Figure 7: Reflection

In figure 7, the image if each point is on a line perpendicular to the line of reflection, and the same distance away as the original point.

Reflections in space take place about planes. Again, if you draw a line from a point on the figure perpendicular to the plane and go the same distance on the other side, you find the image of original point. Reflections in space can be illustrated by mirrors; instead of a line of reflection, it's a plane of reflection.

Rotations

A **rotation** can be described as turning a figure around a point. The direction and degrees of rotation should be specified. (Clockwise is the right; counterclockwise is to the left.) The point is called a center of rotation. If you draw a line from a point on the figure to the point of rotation, then draw in the specified angle, the image is the same distance along the second leg of the angle as the original point was from the point of rotation. (The center of rotation is the only fixed point.)

 *Figure 8: Rotation* https://en.wikipedia.org/wiki/Rotation#/media/File:Rotation_illus.svg 	 *Figure 9: Rotation* https://commons.wikimedia.org/wiki/File:Rotation_illustration2.svg

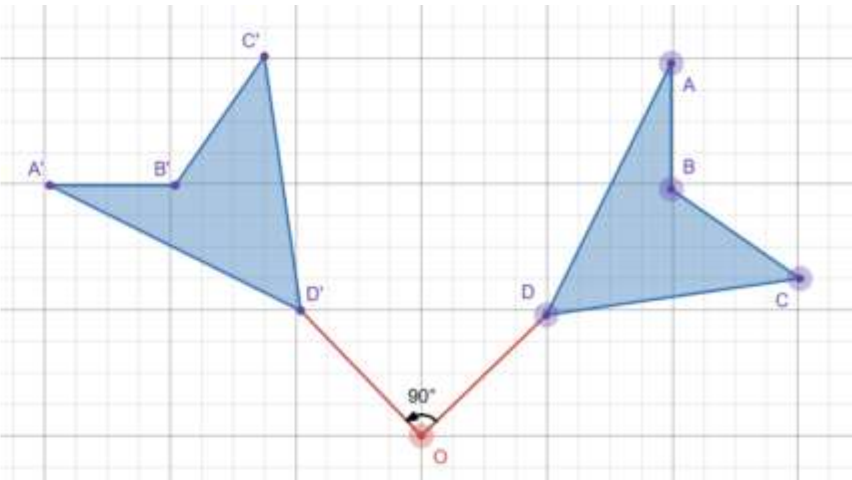


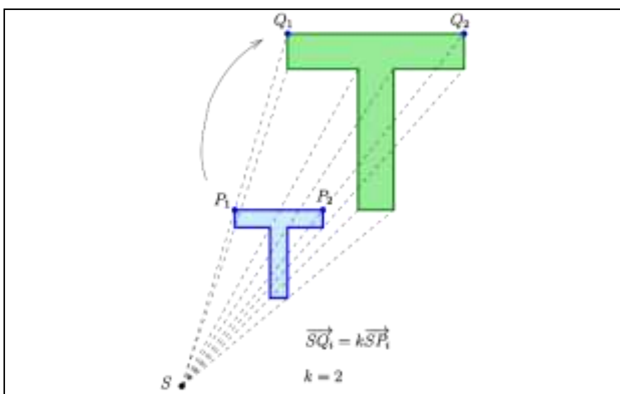
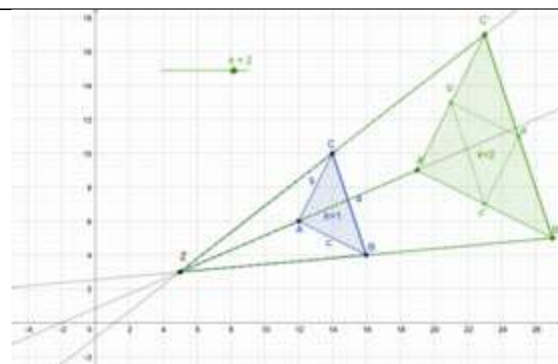
Figure 10: Rotation 90 degrees counterclockwise around point O

In Figure 10, the quadrilateral is rotated 90° clockwise around point O. If you a draw a line from each point to O, then construct a right angle and go the same distance, you get the image.

Figures in space are rotated around lines.

Dilation or Scaling

Dilation, or scaling isn't always included in transformations, because the size of the shape isn't maintained. The image is the same shape, but the size changes, in proportion to the original figure. A dilation that produces a larger image is an **enlargement**; a smaller image is a **reduction**. A description of a dilation includes a **scale factor** (proportion) and the center of dilation. (If scale > 1, it's an enlargement; if 0 < scale < 1 it's a reduction.) The image is *similar* to the original figure; angles are congruent and sides are in proportion.

 *Figure 11: Dilation by 2* *https://en.wikipedia.org/wiki/Homothety#/media/File:Zentr-streck-T-e.svg* 	 *Figure 12: Scaling by 2* *https://commons.wikimedia.org/wiki/File:Zentrische_Streckung_mit_k%3D2.png*

If the center of dilation is the origin, then you multiply each vertex of the shape by the scaling factor. If the center of dilation in NOT the origin, you must measure the distances from each vertex to the center of dilation and the multiply by the dilation factor (distance from the center of dilation, not the origin).

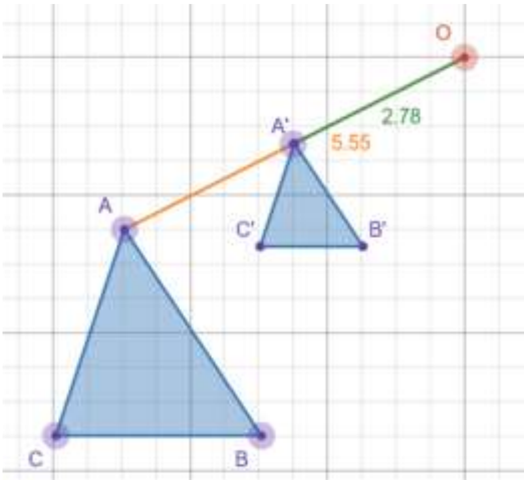


Figure 13: Scaling by 1/2 with center O

In figure 13, the triangle is scaled by ½ with center O. Each image is 1/2 the distance from the center as the original point. (The orange line is the distance from A to O; it overlaps the distance in green.)