NUMBERS IN BASE 5

Base 5 Number System

There's nothing magical about base 10; it's just what we're the most familiar with. We'll be using base 5 as our primary example   (Think about why base 10 is natural. Why do you think base 5 might make sense?) We'll be using base 5 to illustrate the structure behind place value and operations. You've been using base 10 your whole life; switching bases helps you see the places where students might run into trouble because you struggle in base 5.

In base 5, we can write any number, as big or small as we want, using only 5 digits: 0, 1, 2, 3, 4. We can tell the size of the number by the position it's in.

Consider **14302 (base 5)**.   The 4 doesn't stand for just 4, it's really 4 x 125 because of its position.

1	4	3	0	2
5^4	5^3	5^2	5^1	5^0
625	125	25	5	1
Stack of Cubes	Cubes	Flats	Longs	Units

Keep in mind I can keep adding powers of five and get as big as I want, or I can add a decimal point and go down to fifths (5^{-1}), twenty-fifths (5^{-2}), one twenty-fifths (5^{-3}), etc., which are just smaller powers of 5. As you go up each place value you multiply by five (as you go down, you divide by five).

Base 5 Blocks

We can model numbers in base 5 with **Base 5 Blocks**. They look a lot like base 10 blocks; they're just set up in groups of 5 instead of 10.

There are blocks for ones, fives, twenty-fives, and one twenty-fives. The idea can be extended, but those are the most common ones used.

Ones: commonly called **units**. Really just a single cube.

Figure 1: Unit
https://openclipart.org/detail/204860/decimal-system-1
CC0 1.0

Fives: commonly called **longs**. Five units connected.

Figure 2: Long

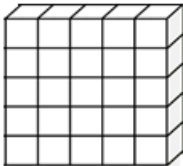**Twenty-fives:** commonly called **flats**. Five longs connected.

Figure 3: Flat

One twenty-fives: commonly called **blocks** or **cubes**. Some resources make a flat printable version called a **long flat** (5 flats connected in a line). Five flats stacked.
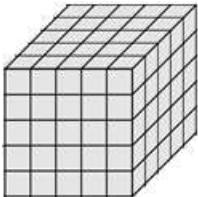

Figure 4: Cube

You can extend the idea: the next size up would be five blocks stacked together (block long?), then five of those (block block?), etc.

Realistically, it's easier to simplify and just draw 2 dimensional versions. You do need to know what base you're working with, though—the simplified images are the same regardless of the base.
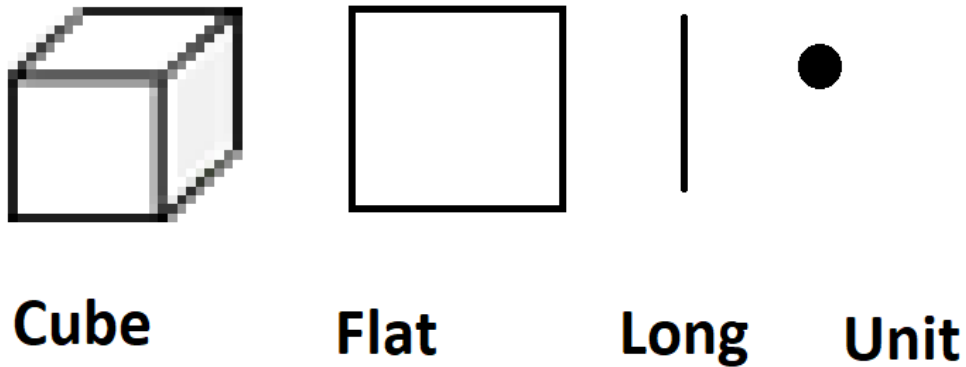

Cube   **Flat**   **Long**   **Unit**

Figure 5: Simplified Base Blocks

To represent a number using base 5 blocks, you simply include the same number of each type as block as the digit associated with that place value.

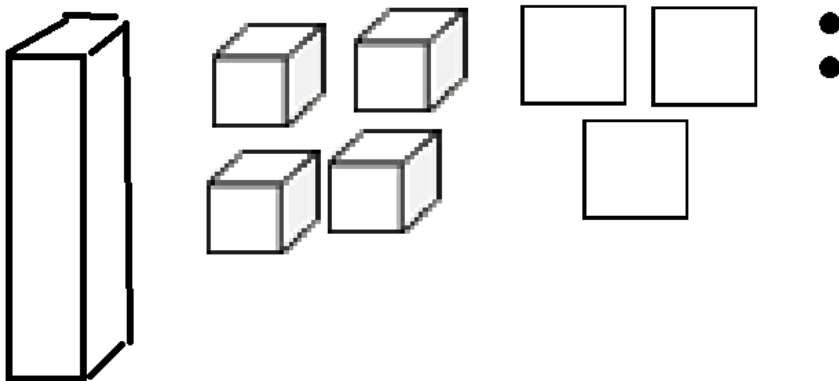$14302_{(base\ 5)}$ **1 cube stack, 4 cubes, 3 flats, no longs, and 2 units**

Figure 6: 14302 (base 5) in Base 10 blocks