NUMBERS IN ANY BASE

Base *b* Number System

There's nothing magical about base 10 or 5. We can write numbers using any base we want. The structure is exactly the same.

In base 5, we can write any number, as big or small as we want, using only 5 digits: 0, 1, 2, 3, 4, up to *b-1*. We can tell the size of the number by the position it's in.

Consider **14302 (base *b*)**. The 3 doesn't stand for just 3, it's really $3xb^2$ because of the place it's in. Note: in this example, *b* can be any number bigger than 4 (because we used that digit).

1	**4**	**3**	**0**	**2**
b^4	b^3	b^2	b^1	b^0
bxbxbxb	bxbxb	bxb	b	1
Stack of Cubes	Cubes	Flats	Longs	Units

Keep in mind I can keep adding powers of five and get as big as I want, or I can add a decimal point and go down to 1/bs (b^{-1}), $1/b^2$s (b^{-2}), $1/b^3$s (b^{-3}), etc., which are just smaller powers of *b*. As you go up each place value you multiply by *b* (as you go down, you divide by *b*).

Base *b* Blocks

We can model numbers in base *b* with **Base 5 Blocks**. They look a lot like base 5 or base 10 blocks; they're just set up in groups of *b* instead of 5 or 10.

There are blocks for ones, bs, b^2s, and b^3s. The idea can be extended, but those are the most common ones used.

You can extend the idea: the next size up would be *b* blocks stacked together (block long?), then *b* of those (block block?), etc.

We can use the same simplified image for any base.

Realistically, it's easier to simplify and just draw 2 dimensional versions. You do need to know what base you're working with, though—the simplified images are the same regardless of the base.

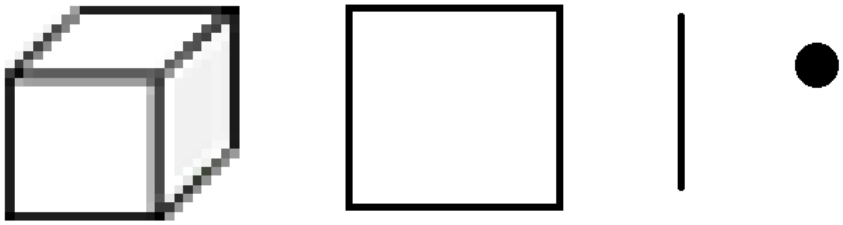

Figure 1: Simplified Base Blocks

Base 2 is often called **binary** and only uses 0's and 1's.

Base 8 is **octal**. It uses 0-7.

Base 16 is **hexadecimal**. It uses 0-9, then a single letter to represent 10-15. Computer programmers tend to use lowercase letters, but this book will use upper case.

You can literally use any base you want, as long as you define the "digits" that represent the numbers bigger than 9. You could use upper case, lower case, and even greek symbols if you had a high enough base.

Base 10 Number	10	11	12	13	14	15	16	17	18	19	20	21	22
Symbol in Other Bases	A	B	C	D	E	F	G	H	I	J	K	L	M
Base 10 Number	23	24	25	26	27	28	29	30	31	32	33	34	35
Symbol in Other Bases	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Examples:

1. **346 (base 7)** stands for $\mathbf{3 \times 7^2 + 4 \times 7 + 6 = 181}$ in base 10.

Note: 346 can't be a base 5 number, because we use the digit 6. It can be a number in any base bigger than 6. That's why it's important to denote the base if it's not 10.

2. **11011 (base 2)** stands for $\mathbf{1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1 = 31}$ in base 10.

Note: 11011 actually be a number in any base.

3. **CAB (base 12)** stands for $\mathbf{12 \times 12^2 + 10 \times 12 + 11 = 1859}$ in base 10.

4. **CAB (base 15)** stands for $\mathbf{12 \times 15^2 + 10 \times 15 + 11 = 2861}$ in base 10.