MULTIPLYING AND DIVIDING FRACTIONS

Multiplying Fractions

If we're multiplying a fraction by a whole number, we can use our traditional strategy: make the given number of copies of the fraction, then combine them to see what the total is.

Consider $\frac{5}{7} \times 3$:

We make 3 copies of 5/7. Then we combine them into one set of fraction bars, filling in any holes. We see that this gives us 15/7 or 2 1/7.

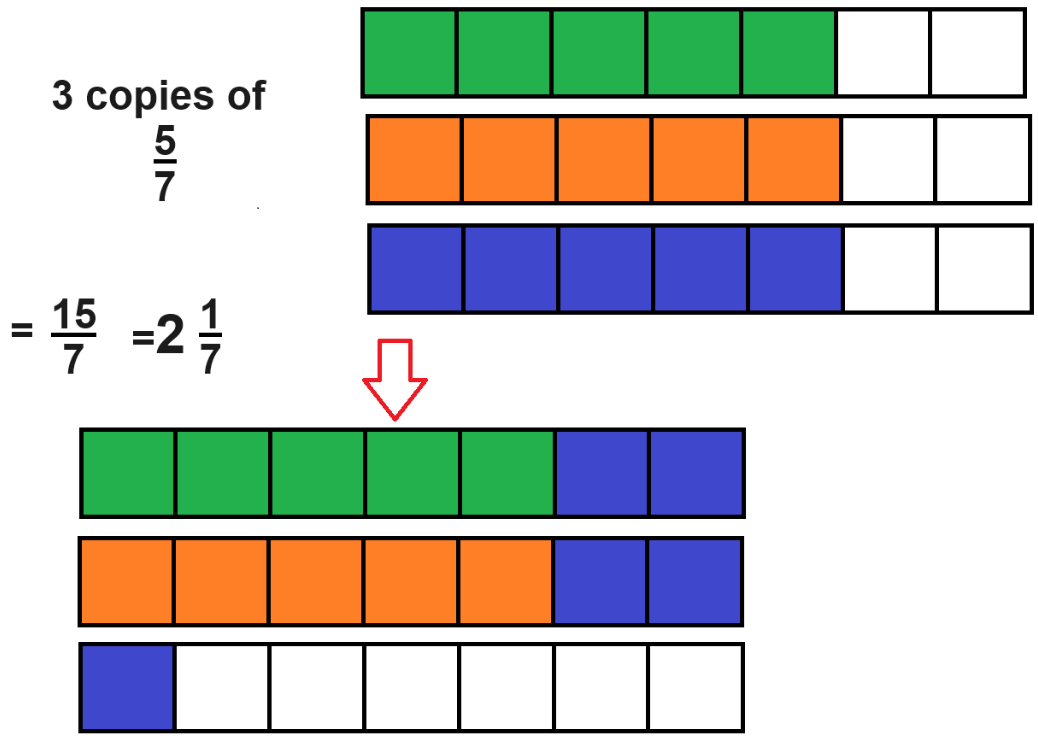


Figure 1: Constant times a fraction

But what happens if both the factors are fractions? You probably learned something like "multiply straight across," but how do we see that in a model? Recall that we can interpret 3x5 as 3 rows of 5 in an array model (we did this with integers). We can do essentially the same thing with fractions. Represent one fraction vertically and one fraction horizontally and see the overlap. This is called an **area model.**

Consider $\frac{2}{3} \times \frac{3}{4}$:
Represent 2/3 vertically and ¾ horizontally. Look for the overlap. We generally do this in a single box, but for the first illustration let's see the two fractions separately first.

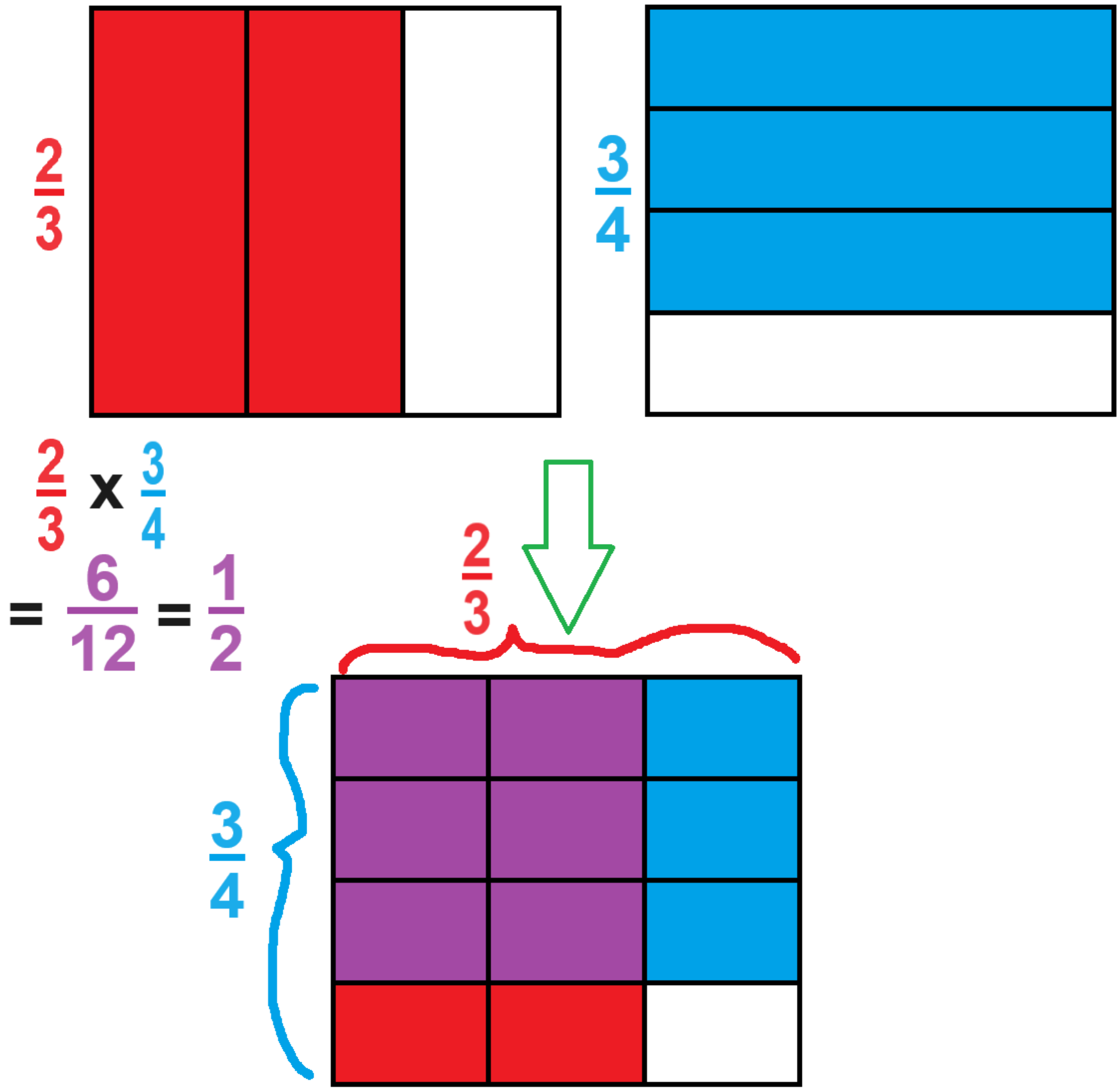


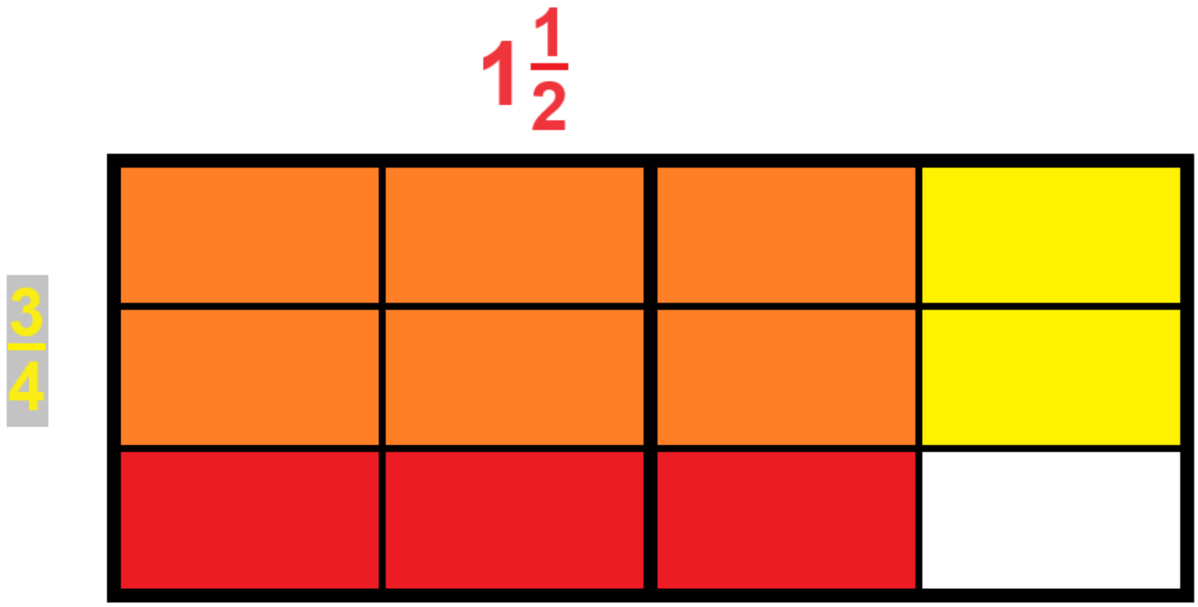
Figure 2: Multiplying fractions with an area model

This model is very nice when you can color the sections with highlighters and can actually see the colors overlap.

Note that this model explains why you can "multiply across"; it's showing how the denominators contribute to the total number of boxes that the unit is split into. The numerators are represented by the shaded portions.

You can extend this model to fractions bigger than one, but you have to be clear where the unit (or one) is represented.

In the example below, we're multiplying 1 1/2 by 3/4. Note the heavier border around outside; this represents our unit. So 1 whole box is shaded vertically, and half of the second, for 1 1/2. 3/4 is shaded horizontally. Note that the colored sections extend across both bars. This is because we're multiplying 3/4 by both 1 and 1/2.



$$1\frac{1}{2} \times \frac{3}{4} = \frac{6}{6} = 1$$

Figure 3: Multiplying fractions bigger than 1

Dividing Fractions

When we ask for 15÷3, what are we thinking? How many times does 3 fit into 15? We can do something similar with fractions.

Let's consider $\frac{3}{4} \div \frac{1}{2}$. How many copies of 1/2 fit inside 3/4?
If you find a common denominator, it's easier to see. So 3/8 is 6/8 and 1/2 is 4/8. How many copies of 1/2 fit inside 3/4? This is illustrated with the lighter green—1 copy of 1/2 (4 blocks) fit inside the shade region of 3/4, with 2 out of 4 blocks left over. So the answer is 1 2/4 or 1 1/2.

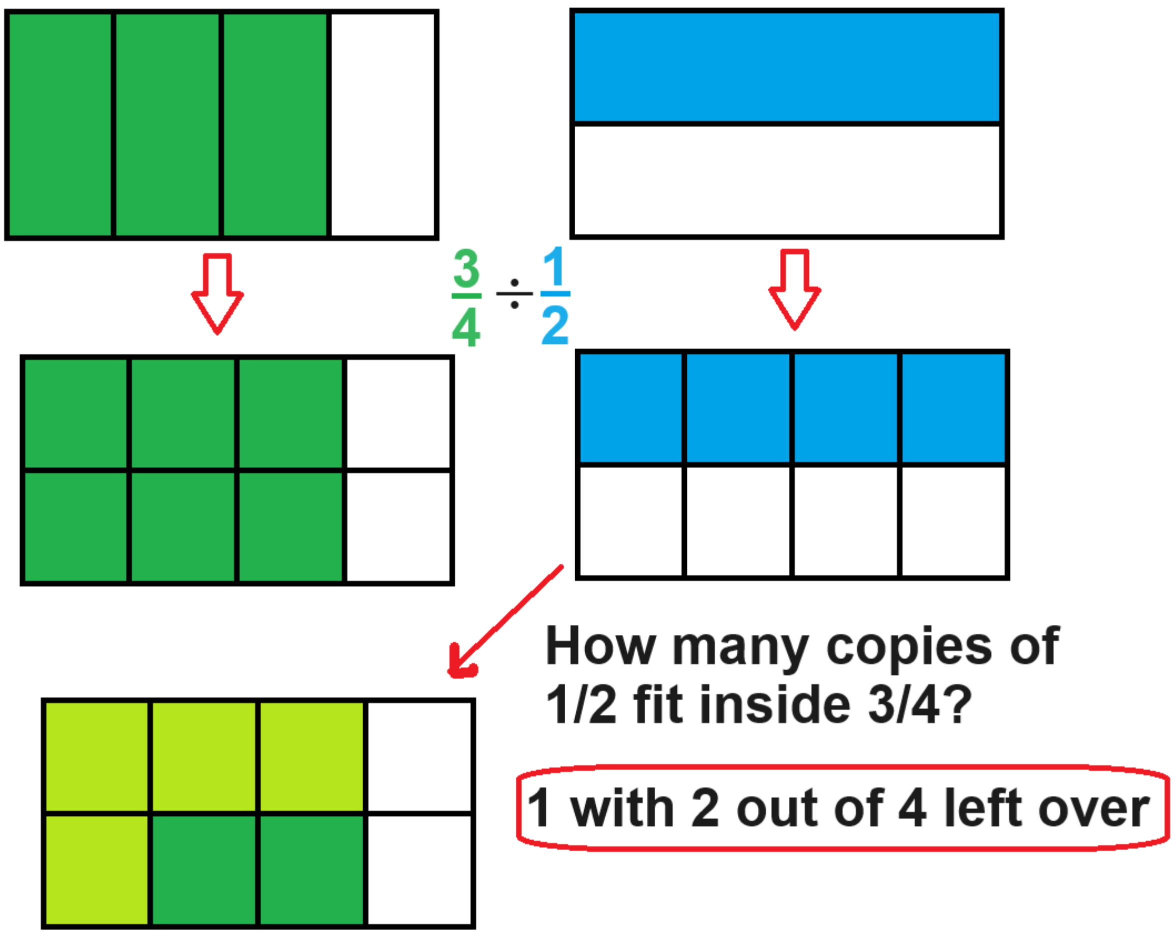


Figure 4: Dividing fractions with a model

Let’s try another example. $\frac{\mathbf{2}}{\mathbf{5}} \div \frac{\mathbf{1}}{\mathbf{2}}$

This time our common denominator is 10. So 2/5 is 4/10 and 1/2 is 5/10. How many copies of 1/2 fit inside 2/5? This time the whole thing doesn’t fit, so 4 out of 5 pieces do. So 2/5 divided by 1/2 is 4/5.

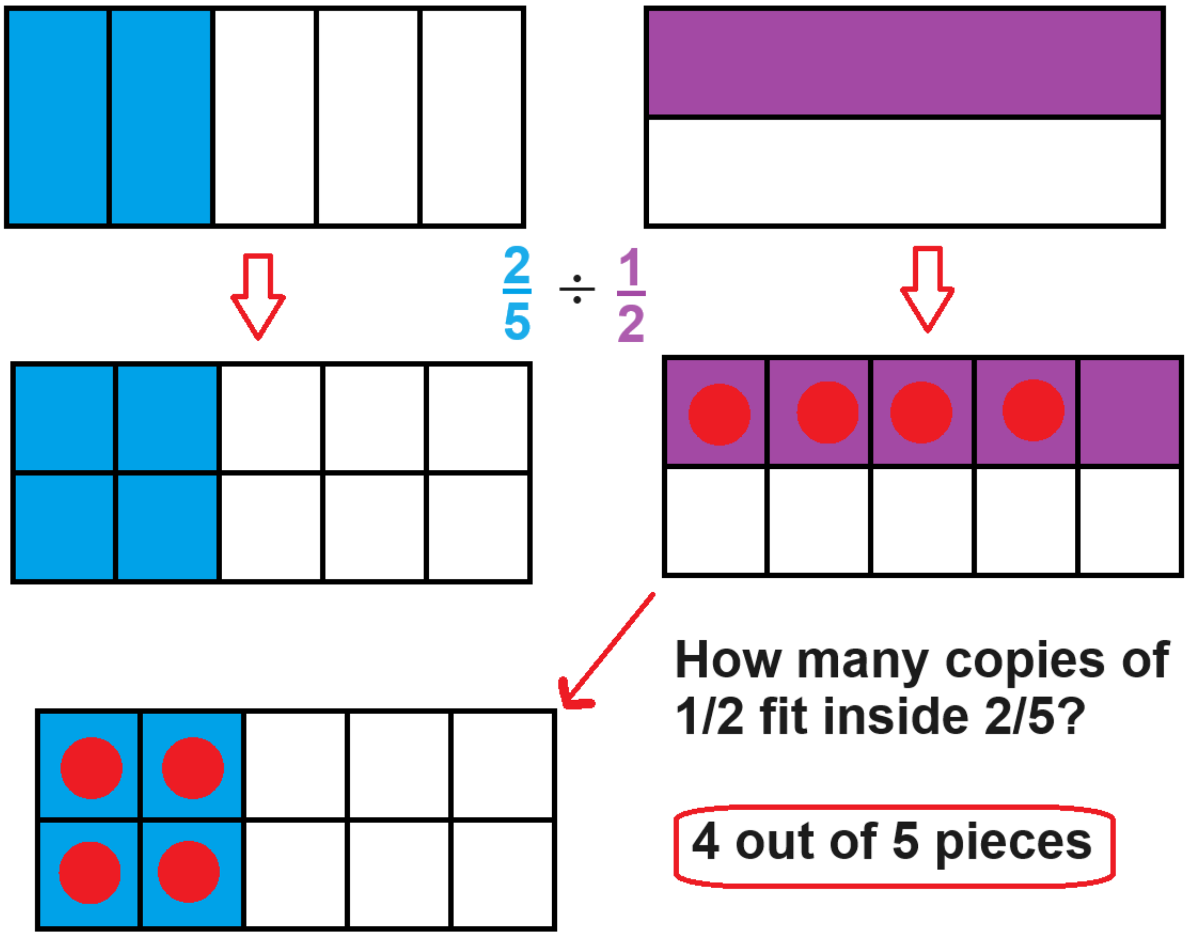


Figure 5: Dividing fractions with a model, take 2

Okay, but how does that tie into the standard algorithm? (“Keep change flip”, where you multiply by the reciprocal of the divisor.) Let’s look at that first example again:

$$\frac{\mathbf{3}}{\mathbf{4}} \div \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{6}}{\mathbf{8}} \div \frac{\mathbf{4}}{\mathbf{8}} = \frac{\mathbf{6}}{\mathbf{8}} \times \frac{\mathbf{8}}{\mathbf{4}} = \frac{\mathbf{6}}{\mathbf{4}}$$

The numerator of the second fraction becomes the denominator of our answer. We’re asking how many of the second fraction’s numerator (bars) fits inside the first fraction’s numerator (after we find a common denominator).

But why can we flip and multiply? What we're actually doing is multiplying by the reciprocal of the second fraction so that that the denominator becomes 1.

$$\mathbf{\frac{3}{4} \div \frac{1}{2} = \frac{6}{8} \div \frac{4}{8} = \frac{\frac{6}{8}}{\frac{4}{8}} = \frac{\frac{6}{8}}{\frac{4}{8}} \times \frac{\frac{8}{4}}{\frac{8}{4}} = \frac{\frac{6}{8} \times \frac{8}{4}}{1} = \frac{6}{8} \times \frac{8}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}}$$

Believe it or not, you can actually divide across, too:

$$\mathbf{\frac{3}{4} \div \frac{1}{2} = \frac{6}{8} \div \frac{4}{8} = \frac{6 \div 4}{8 \div 8} = \frac{6 \div 4}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}}$$

Remember that there's nothing magical about the standard algorithm (or the way you learned to do it). This way is valid, too.