

MULTIPLICATION MODELS

What does **multiplication** really mean? Is the way you learned to add the only way to do it? Do we all have to solve problems the same way?

Consider the following:

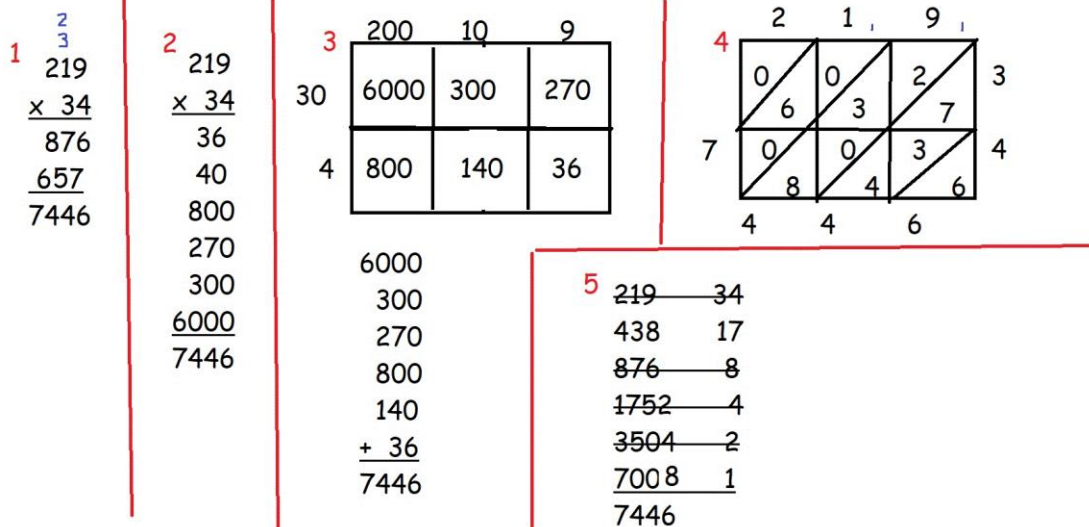


Figure 1: 218x34 worked 5 different ways

It's the same problem worked five different ways. They all got the same answer. Is one more right than the others?

Take a minute and try to figure out what's going on for each before you read the rest of the section. Some of these you may have seen, some may be totally new. All are equally valid and several have historical significance.

1 Standard Algorithm

This is probably the most familiar. You're multiplying 4×9 , writing down 6 and carrying the 3, then $4 \times 1 + 3$, then 4×2 . Then you shift over a place. **Why?** Some people put in a zero as a placeholder. **Why?** Both are because we aren't really multiplying by 1, we're multiplying by 10. The space or zero indicates the shift in place value. Then we do 3×9 (write down 7, carry the 2), $3 \times 1 + 2$, 3×6 , and add to get the final answer. Did you every stop to think about why we do it this way? How would you explain it to someone who's never seen it before?

2 Partial Products

This time, instead of carrying, the student is writing down the answer for each multiplication. $4 \times 9 = 36$. $4 \times 10 = 40$, etc. Notice the place value is incorporated in the final answer.

3 Area Model

Break down the numbers into their place values. $219 = 200 + 10 + 9$; $34 = 30 + 4$. Draw a box with one square for each place value—so 3×2 , and write the expanded numbers around the box (one number vertically and one horizontally). Multiply the top and side of each box. Then add the answers in the middle.

4 Lattice

You will need a box that matches the dimensions of your number. 219×34 needs a 3×2 box because there are 3 and 2 places, respectively. Put one number on top and one down the right, one digit per box. Multiply each box and separate the answer by place value. Note that if the answer is a single digit, you add a 0 to the tens place. Then add down the diagonals to get your answer.

5 Russian Peasan

Create two columns, starting with the numbers in the problem. In the second column, half the number. If there's a remainder, ignore it. Go until you get to 1. In the first column, double the number. Then, cross out the rows that have an EVEN number in the second column, and add the remaining numbers in the first. **Why does it work?** This has a connection to the answer in binary (base 2)—the numbers you add are the 1's and the ones you cross out are the 0's.

So Which is Right?

All 5 methods are valid. You could have students in your class using any (or all) of them, and I may not have included them all. Our goal is not just to help students get the answer, but to be comfortably thinking mathematically and to understand the underlying concepts. Algorithms can be forgotten—but if you know the concept, you can recreate the algorithm.

At its base, multiplication is repeated addition. You can model 4×3 by making 3 copies of 4.

At its base, division is about splitting things into groups. You can model $12 \div 3$ by taking 12 things and either splitting it into 3 groups (the answer is how many per group) or splitting into groups of 3 (the answer is how many groups).

Parts of a multiplication problem:

multiplicand x multiplier = product OR factor x factor = product

Parts of a division problem:

dividend \div divisor = quotient + remainder/divisor