INTEGERS

Integers are whole numbers that can be positive, negative, or zero. Students will probably encounter the concept of negative numbers well before they do so formally in school. Examples include money (debits), temperature (degrees below zero), altitude (feet below sea level), and sports (score under par in golf, yardage in football).

Modeling Integers

One common model for integers (which is not emphasized in this section) is on the number line. It's important for students to understand how the integers are mirror-images around zero, a negative number is *always* smaller than a positive number, and that the more negative a number is, the *smaller* it actually is (-15 is smaller than -2, even though 15 is larger than 2). Working with numbers on a number line can help stress these facts that are often confusing. Note that all the operations we're going to discuss for integers can be modeled with a number line—and you may end up teaching it using a number line.

The model we're going to focus on uses two-color counters. The counter are traditionally red on one side and white or yellow on the other. It's important to note that the colors aren't important as long as everyone agrees what the colors stand for. You can also model using 2 different colored counters if the double-sided ones aren't available.

Traditionally, red is considered the negative side. If you don't have access to color, you can add a **+** or **–** inside the circle to indicate which it represents.

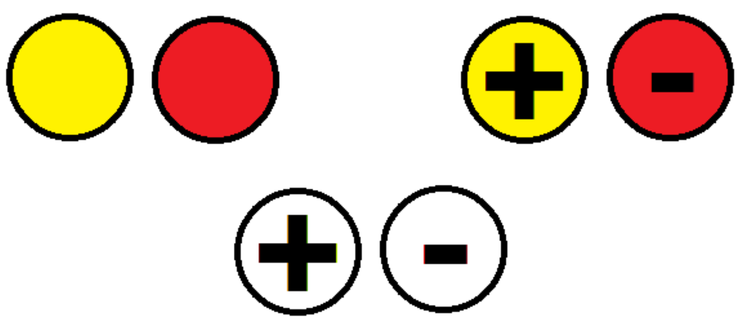

Figure 1: Three Ways to Draw Two Color Counters

We can model addition, subtraction, multiplication, and division of integers using the counters. The most important concept is the idea of a **zero pair.** A positive and a negative together cancel each other out and make zero.

What number is being represented in the image below? Mark through or physically remove any zero pairs and see what's left. (The answer is in the caption. Try it before you peek!)

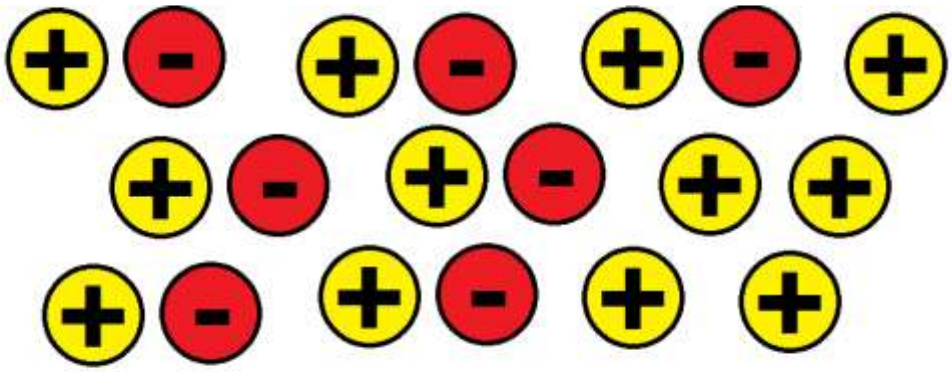

Figure 2: When you remove the zero pairs, this represents 5

Adding Integers

The model for adding integers is similar to what we've done before. If both numbers have the same sign, it's exactly the same. Make a group for each number, then combine them.

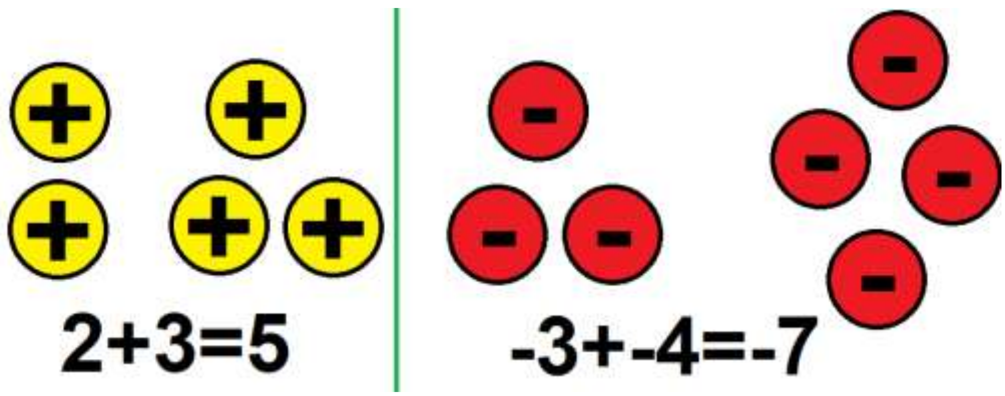

Figure 3: 2+3=5 and -3+-4=-7

When the two numbers being added have different signs, we still make two piles and combine them. Then we have to account for zero pairs to see what the answer is. First, represent the two numbers:

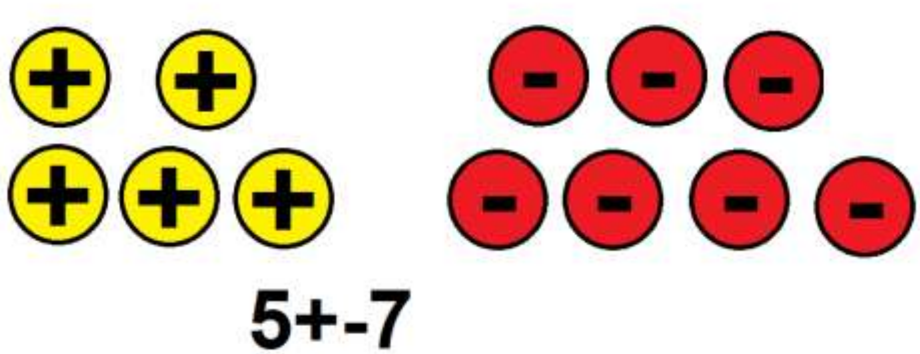

Figure 4: Represent the numbers

Then cancel out any zero pairs (one positive, one negative). The remaining counters should be all the same color, and the number remaining (and their color) give you the answer.

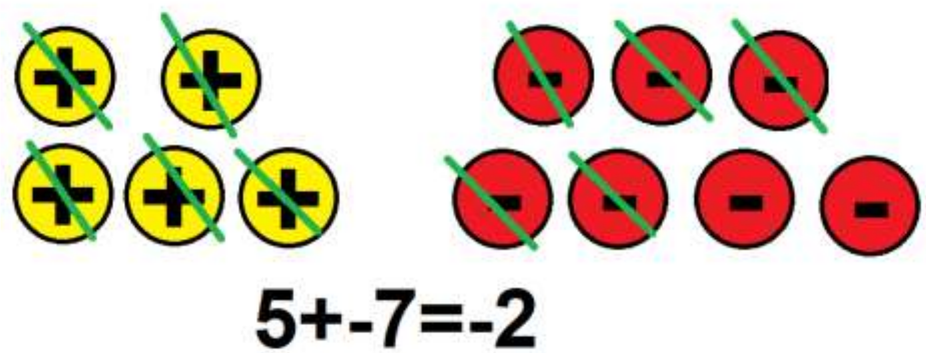

Figure 5: Cancel zero pairs

I recommend having students model several examples, then ask them to determine the rules for adding integers. Help students use careful language and math terminology to come up with a formal statement. It will be easier to remember (or recreate) because they discovered it themselves.

Subtracting Integers

A common model for subtracting integers is **add the opposite**. This is a mathematically sound model that is often explained with number lines. You may end up teaching it, but I find that **take away** is a more natural model, so that's the one this section will focus on. It's also the same model we used with whole numbers earlier, so there's a natural extension.

Start with problems where the numbers have the same sign. Represent the first number, then take away the second. What's left is your answer.

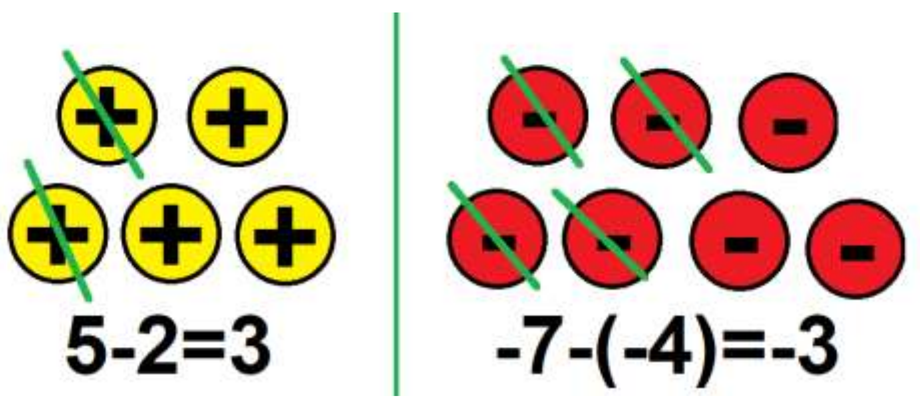

Figure 6: Subtracting integers with same sign

What if the two numbers have different signs? Consider **8-(-5)**. We start the same way, by representing the first number.

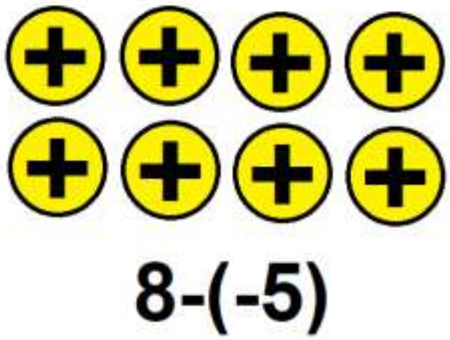

Figure 7: Start by representing the first number

We want to **take away** -5, but we don't have any negatives. What can we add to 8 that doesn't make it a different number? We can add zero. The only way to add more tiles without changing the number we're representing is by adding zero pairs. Add zero pairs until there are at least 5 negatives (you can add more than 5, but then you have more to adjust at the end.

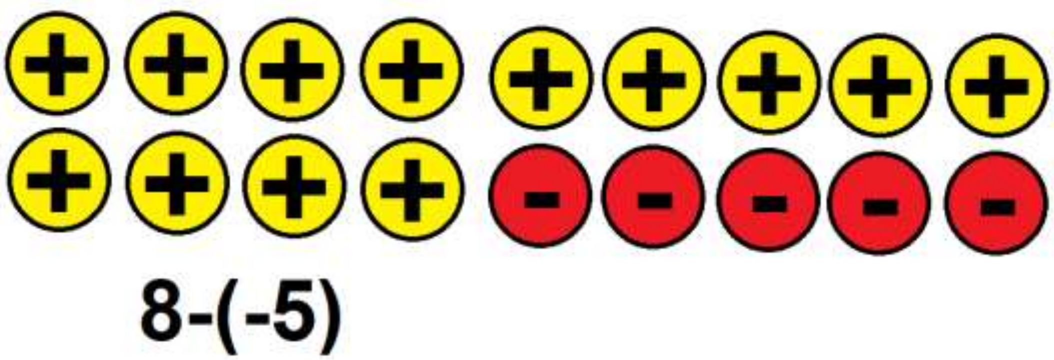

Figure 8: Add zero pairs

Finally, we remove 5 negative tiles. What's left is our answer.

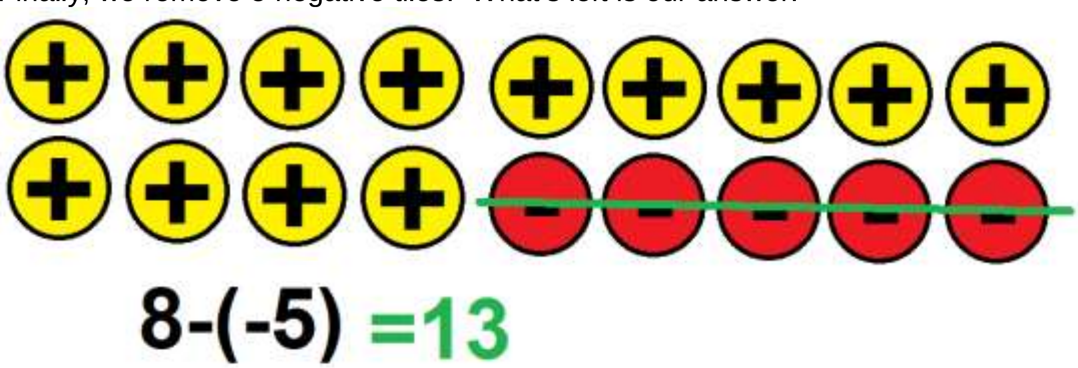

Figure 9: Subtracting -5

Again, you'd do several examples like this, then ask students to define the rule. The model is the same whether we're subtracting a positive or a negative number. (Note that you get the same answer by add the opposite—so that's one way you can check your work if this model is confusing.)

Multiplying Integers

The most straightforward model for multiplying integers is an **array model**. Earlier, we envisioned 3x4 as creating 3 groups of 4. We can also do this as 3 rows of 4.

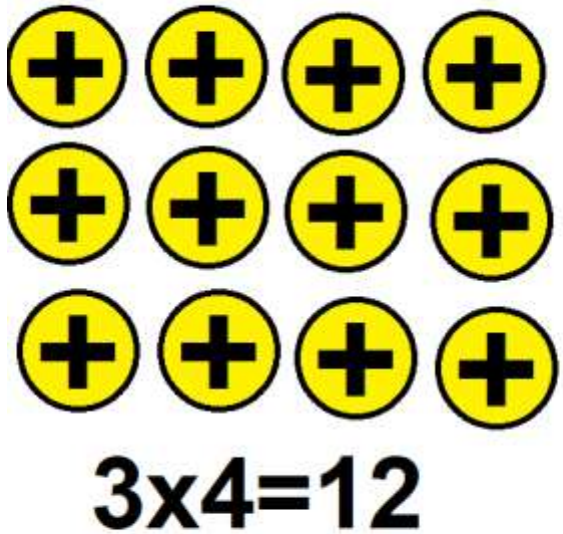

Figure 10: 3x4 using an array model

This interpretation makes sense as long as the first number is positive. But how do we do -3x4? Does -3 rows even make sense? The work around is to represent the problem first as if every number is positive. Then, for every negative, you flip the counters over. Note that a flip turns a positive to a negative, and a negative to a positive.

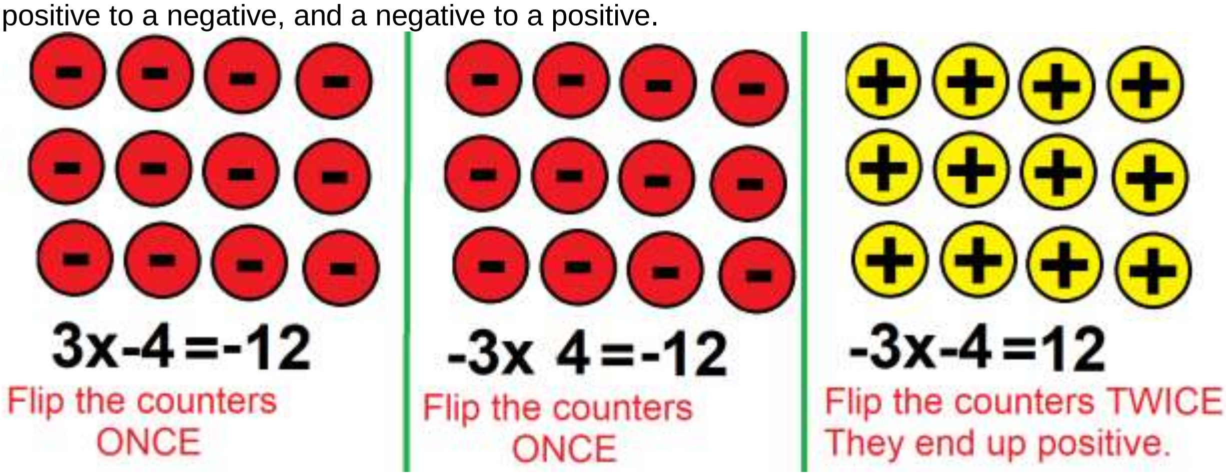

Figure 11: Three problems with flipped counters

Dividing Integers

We model division of integers as the inverse of multiplication, so we use the same flip to adjust for negatives in the problem. 12÷3 can be modeled by taking 12 counters and splitting them up into 3 even rows. The answer is the number of counters per row. Division problems involving one or more negative number work the same way. Start with positives. Split the number up into equal rows. For each negative in the problem, flip all the counters.