DIVISION

Like multiplication is repeated addition, division is repeated subtraction. We can model 12 ÷ 3 by either: taking a group of 12 and splitting it into 3 groups, or taking a group of 12 and splitting it into groups of 3.

Taking a group of 12 and splitting it into 3 groups is an example of a **partitive** interpretation of division. We're taking our number and splitting it into equal size groups. The answer is the number of things in each group.

Taking a group of 12 and splitting it into groups of 3 is an example of the **measurement** interpretation of division. We're taking out number and splitting it into groups of a given size. The answer in the number of groups.

The answer is the same, whichever way you interpret the problem, but this can be confusing to students because the concepts are very different.

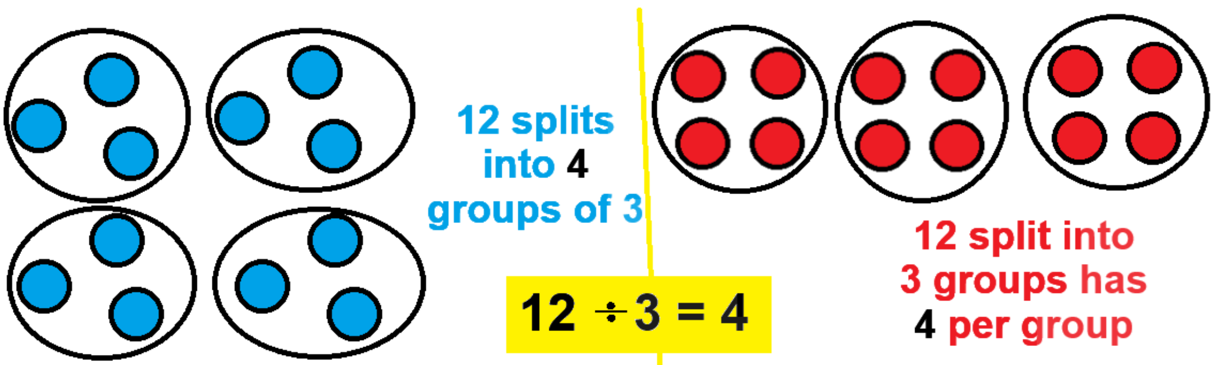

Figure 1: Two models for 12 divided by 3 using colored counters

When we move up to larger numbers, the concept is the same. If we use base 10 blocks, we can handle larger numbers.

Division in Base 10

Consider **247 ÷ 3.**

First, represent the numbers:

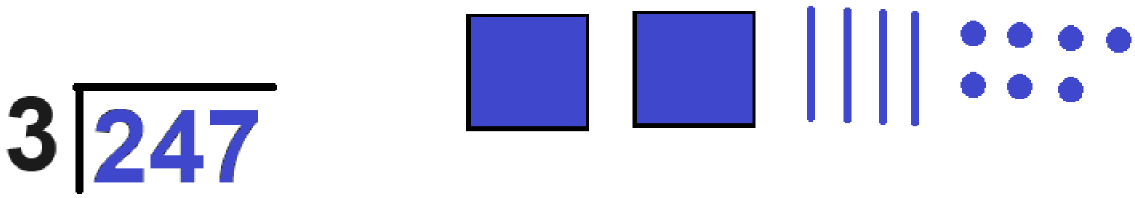

Figure 2: 247 represented with Base 10 blocks

We’re literally going to split 247 into 3 groups. If we can’t split a place value evenly, we regroup the remainder down to the next size block. This is typically done from left to right, which conveniently matches the traditional algorithm for long division.

I can’t split 2 into 3 groups, so I trade each flat in for 10 longs. So now there are a total of 24 longs.

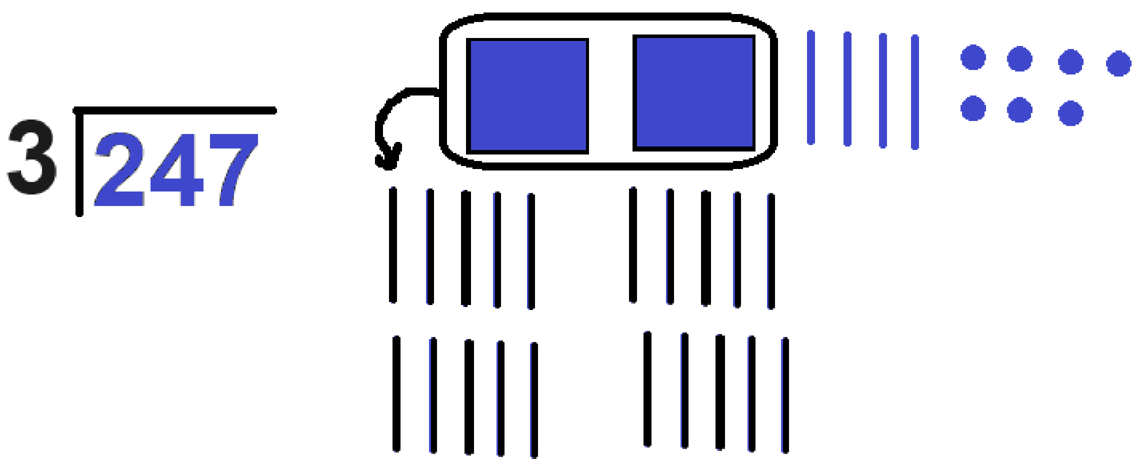

Figure 3: Trade 2 flats for 20 longs

Then, split your 24 units into 3 equal groups—with 8 in each group. 3x8=24, so we used all our longs. If there were any longs left, our next step would be to trade them in for units (bring down the 7 represents our transition to the next place value).

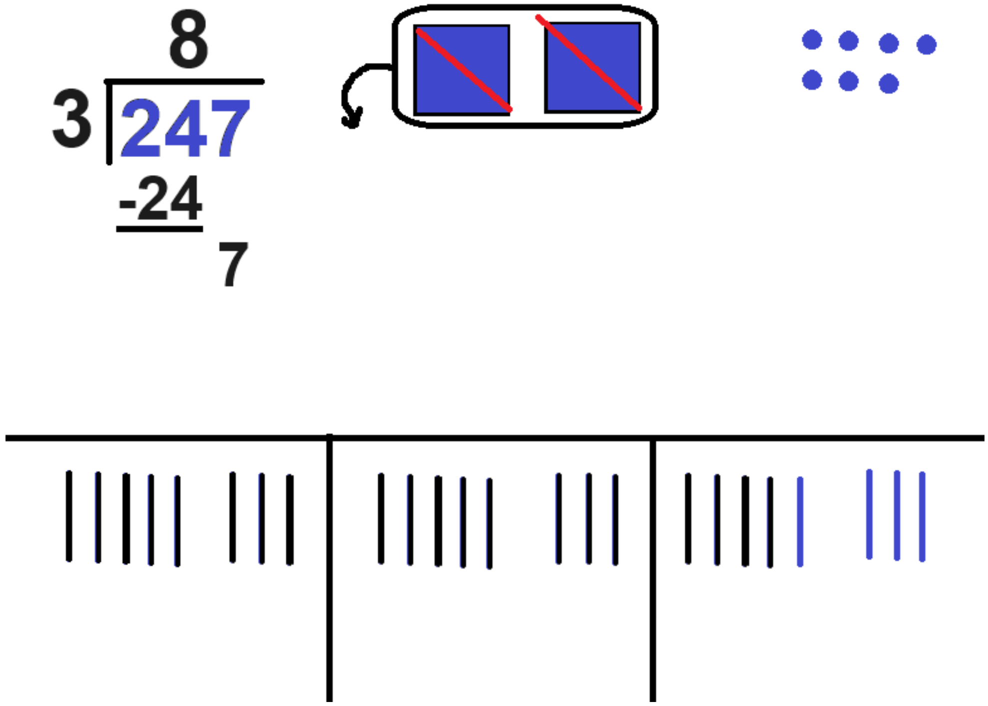

Figure 4: Splitting 24 longs into 3 groups

Now we split our units into 3 groups. There will be 2 in each group with 1 left over—our remainder. Our answer is 82 remainder 1.

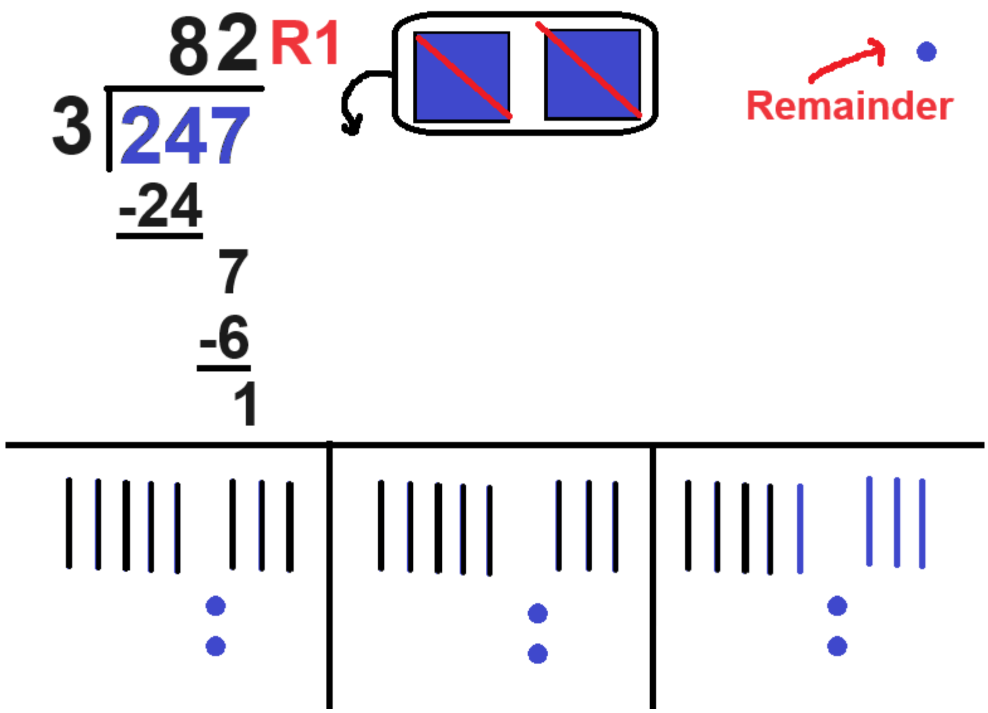

Figure 5: Split 7 units into 3 equal groups; 2 each with 1 reminder

Note that we can extend this concept of division to any number system, and any base. The idea is the same—just split things into the indicated number of groups, and trade in if there are spares.

I could do this in Egyptian hieroglyphs just as easily as I do in base 10.

Here's 572÷2 using Roman numerals:

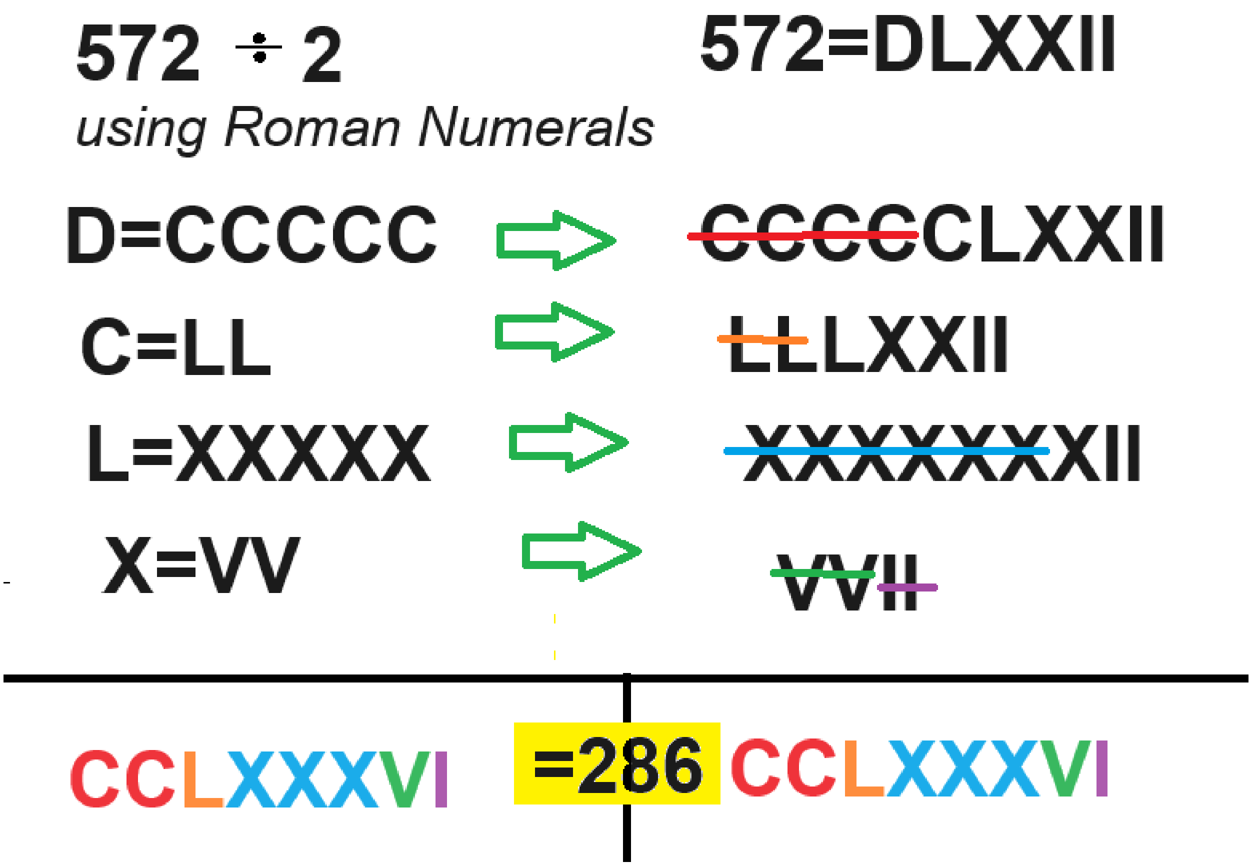

Figure 6: 572 ÷2 using Roman Numerals

Division in Base 5

Division in Base 5 works the same way; all that changes is that we regroup sets of 5 instead of sets of ten.

Consider **243 ÷ 3 (base 5):**

First, represent your number.

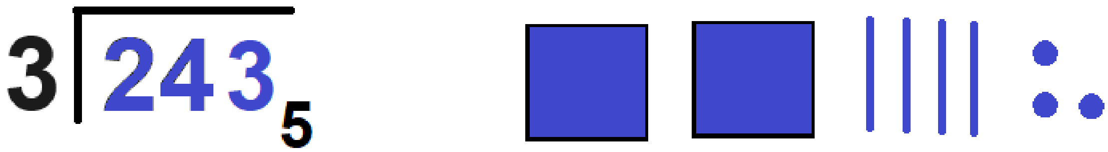

Figure 7: 243 (base 5) represented with Base 5 blocks

Next, start splitting into 3 groups. 2 flats won't split, so trade them down for 5 longs each. We now have 24 (base 5), or 14 longs.

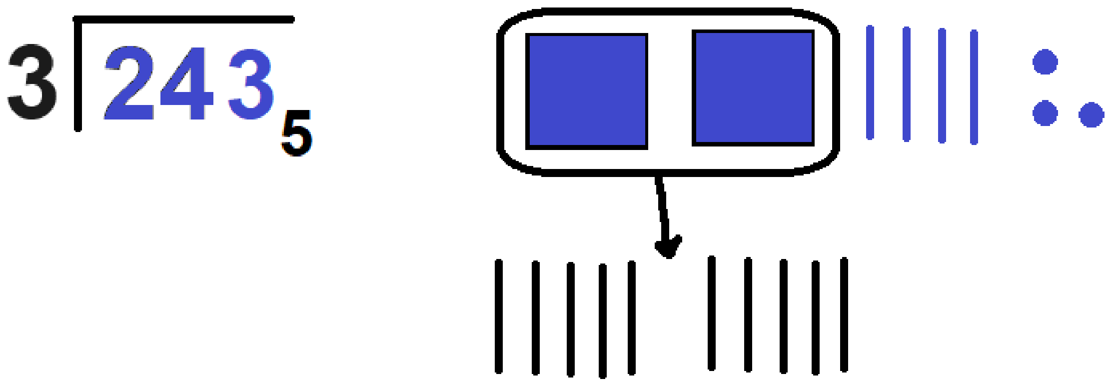

Figure 8: Trade in 2 flats for 5 longs each

Now divide the longs into 3 groups. There will be 4 per group, with 2 left over.

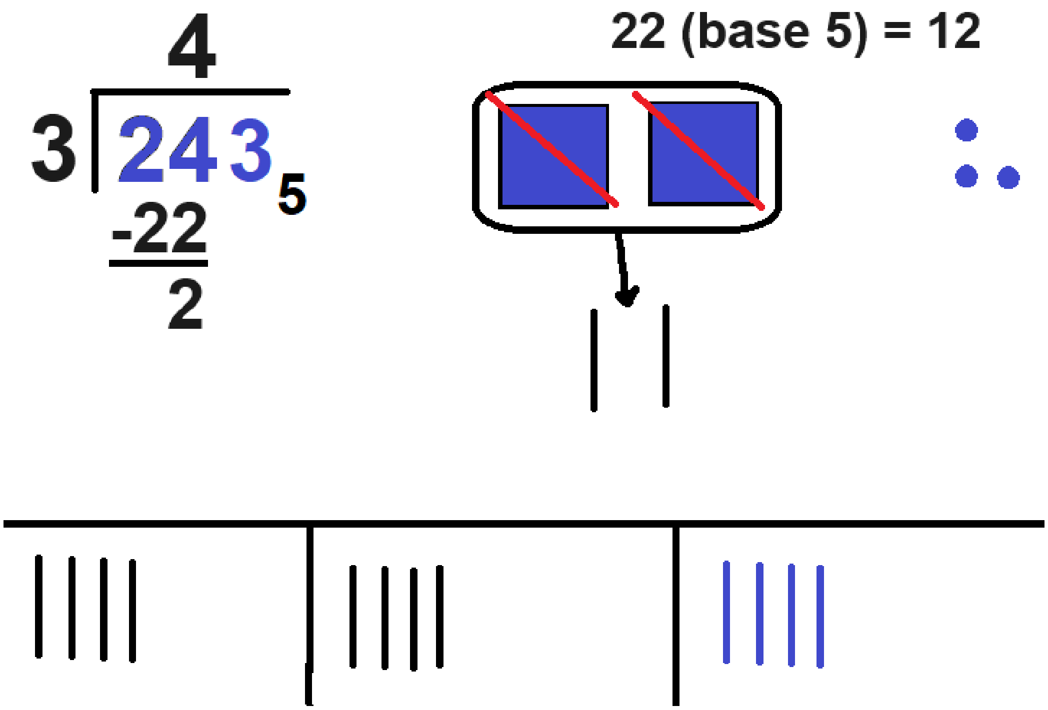

Figure 9: Split 24 longs into 3 groups, with 2 left over

Now trade 2 longs for 5 units each. This gives us a total of 23 (base 5) or 13 units.

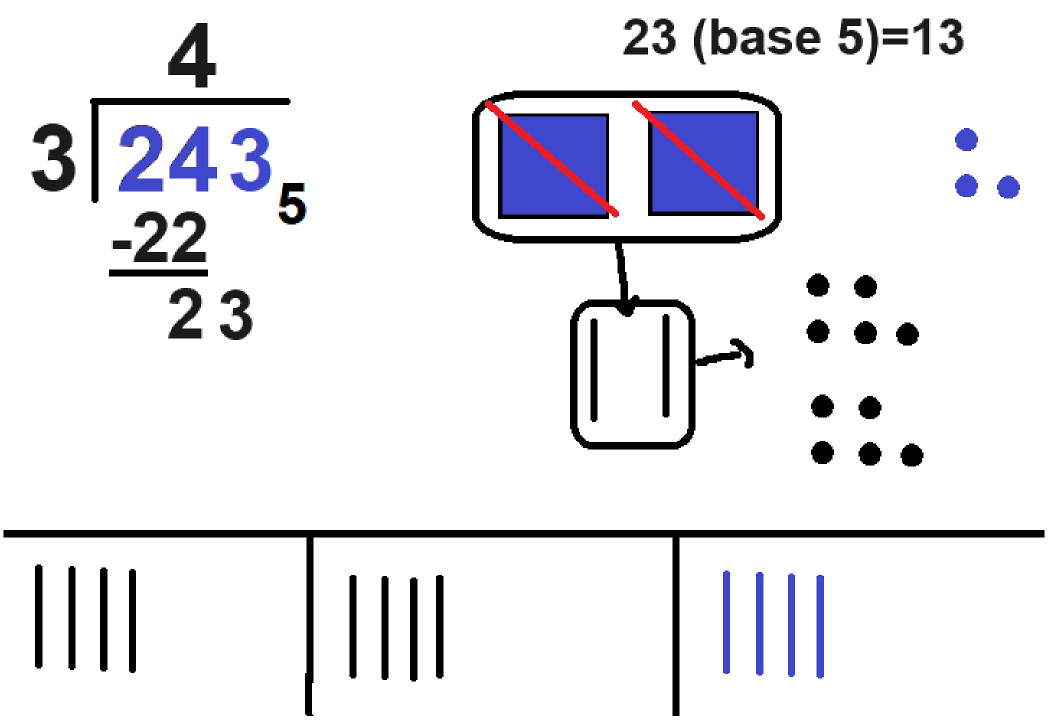

Figure 10: Trade longs for 5 units each

Finally, split the units into 3 groups. There will be 4 each, with one reminder.

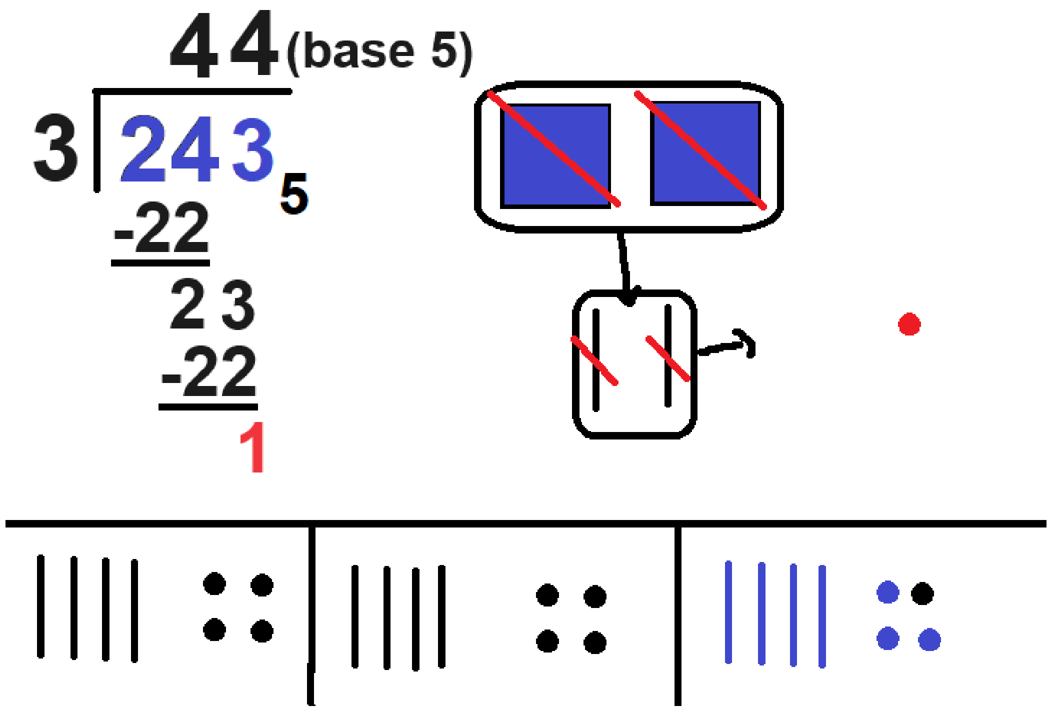

Figure 11: Sorting units

Of course, you would do a lot of examples with models before expecting students to be able to do them on their own. The goal is eventually to be able to do without the model, but the model builds conceptual understanding of what's really going on when we regroup ("borrow" or "carry"). If you forget how to use the algorithm, you can still figure it out if you have the model and conceptual understanding as a foundation.