CONVERSIONS AMONG FRACTIONS, DECIMALS, & PERCENT

We have discussed models and operations with fractions, decimals, and percent separately, but they're really just different representations of the same thing. Think about how similar the models are—that's not a coincidence So, how do we convert from one version of a number to another? Let's consider the 6 types (in alphabetical order):

Decimal to Fraction

To convert a decimal to a fraction, think about how the number is officially read. Not "five point two three", but "five and 23 hundredths." The tail end of the number tells you what the denominator is. Write the decimal part the numerator over an appropriate power of 10, then simplify the result. (Note: the appropriate power of 10 has as many 0's as the decimal has places past the decimal point.)

D->F 1:

$$5.23 = 5\frac{23}{100}$$

D->F 2:

$$0.2725 = \frac{2725}{10000} = \frac{109}{400}$$

Repeating Decimal to Fraction

If you have a repeating decimal, you can convert it to a fraction, too. The secret is to note how many decimal places repeat. Multiply by an appropriate power of ten, then subtract and divide. If one digit repeats, multiply by 10; if two digits repeat, multiply by 100, etc. The idea is to get the repeating part to cancel out when you subtract. Finally, simplify the fraction if necessary.

D->F 3:

$$0.\overline{8}$$

One digit repeats, so multiply by 10:

$$10x = 8.8888 \ldots.$$
$$-x = 0.8888 \ldots$$

Subtract:

$$9x = 8$$
$$x = \frac{8}{9}$$

D->F3:

$$0.2\overline{35}$$

Two digits repeat, so multiply by 100:

$$100x = 23.53535 \ldots$$
$$-x = 0.23535 \ldots$$

Subtract:

$$99x = 23.3$$
$$x = \frac{23.3}{99} = \frac{233}{990}$$

Decimal to Percent

To convert from a decimal to a percent, remember that percent means “out of 100”. You multiply the decimal by 100%, which has the effect of moving the decimal point 2 places to the right. Note that you can have more than 100%, if the original number is bigger than 1.

D->P 1:	D->P2:
$3.5 = 3.5 \times 100\% = 350\%$	$0.376 = 0.376 \times 100\% = 37.6\%$

Fractions to Decimals

To convert from a fraction to a decimal, divide the numerator by the denominator. Unless told to do so, DO NOT ROUND. Keep dividing until the fraction either terminates (stops) or repeats.

F->P 1:

$$\frac{3}{8} = 3 \div 8$$

```
  0.3 7 5
8| 3.0 0 0
 -2 4
    6 0
   -5 6
      4 0
     -4 0
```

Figure 1: 3 divided by 8

Terminates. Solution is 0.375

F->P 2:

$$\frac{4}{7} = 4 \div 7$$

```
  0.5 7 1 4 2 8
7| 4.0 0 0 0 0 0
 -3 5
    5 0
   -4 9
      1 0
     -7
       3 0
      -2 8
         2 0
        -1 4
           6 0
          5 6
             4
```

Figure 2: 4 divided by 7

Repeats. Solution is $0.\overline{571428}$

Fractions to Percent

We start out the same way we do when converting to a decimal. We don't want to divide all the way to the end, however. Instead, keep in mind our goal: we're converting to a percent. To do that, we're going to multiply by 100% after dividing. So we only need to divide out two decimal places after the decimal point. The remainder is a fraction of the divisor.

F->P 1:

$$\frac{3}{8} = 3 \div 8$$

$$\begin{array}{r} 0.37 \\ 8\overline{)\,3.00} \\ -24 \\ \hline 60 \\ -56 \\ \hline 4 \end{array}$$

Figure 3: 3 divided by 8 after 2 decimal places

Write the remainder as a fraction of the divisor, simplify, and multiply by 100%:

$$= .37\frac{4}{8} \times 100\% = 37\frac{1}{2}\%$$

F->P 2:

$$\frac{4}{7} = 4 \div 7$$

$$\begin{array}{r} 0.57 \\ 7\overline{)\,4.00} \\ -35 \\ \hline 50 \\ -49 \\ \hline 1 \end{array}$$

Figure 4: 4 divided by 7 after 2 decimal places

Write the remainder as a fraction of the divisor, simplify, and multiply by 100%:

$$= .57\frac{1}{7} \times 100\% = 57\frac{1}{7}\%$$

Percent to Decimal

Since percent means "out of 100", to convert a percent to a decimal, you divide by 100, which has the same effect as moving the decimal to the right 2 places.

P->D 1:

$$75\% = \frac{75}{100} = 0.75$$

P->D 2:

$$38\frac{1}{2}\% = 38.5\% = \frac{38.5}{100} = 0.038$$

If your percent has a terminating fraction, go ahead and convert it to a decimal by dividing.

Percent to Fraction

Since percent means “out of 100”, we can convert any percent to a fraction by dividing by 100%.

P->F 1:

$$48\% = \frac{48}{100} = \frac{12}{25}$$

P->F 2:

$$14\frac{2}{5}\% = 14.4\% = \frac{14.4}{100} = \frac{144}{1000} = \frac{18}{125}$$

Convert a terminating fraction to a decimal, divide by 100. You can eliminate the decimal by multiplying numerator and denominator by the same power of 10. Then simplify.

P->F 3:

$$33\frac{1}{3}\% = \frac{100}{3}\% = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100} = \frac{100}{300} = \frac{1}{3}$$

If your percent has a repeating fraction, convert the mixed number to an improper fraction. Then divide by 100 (which is the same as multiplying by 1/100), and simplify.