ALGEBRA TILES

One way to make the abstract concepts of algebra more tangible is **algebra tiles**. We're going to focus on the basic set. There is a large square that's blue on one side and red on the other that represents x^2, a green and red rectangle that represents x, and a small yellow and red square that represents 1. The traditional interpretation is that red means negative.

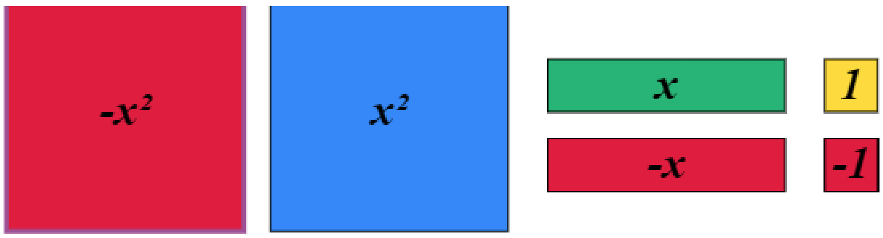

Figure 1: Basic algebra tiles

An expanded set exists that includes different size and color pieces for y^2, y, and xy. There are several variations from different manufacturers, including ones that are three dimensional. You can buy physical sets, print paper sets, find virtual sets, or draw your own (it's just different size boxes). If color isn't an option, you can draw the squares with a + or – in the center to indicate color. All images of tiles in this section originated from https://mathsbot.com/manipulatives/tiles.

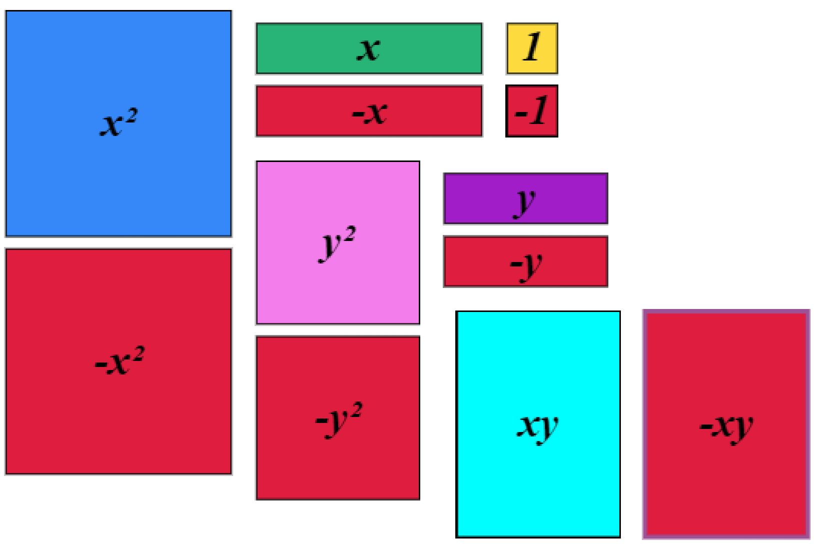

Figure 2: Expanded algebra tiles

Operations with Integers

Operations with integers can be modeled using the small 1 squares. Essentially, these are the same models we did with two-color counters in the Integer section. You can model addition, subtraction, multiplication, and division of integers.

Remember the key concept of the **zero pair**: a positive tile and a negative tile create a zero pair and cancel each other out. Note now, however, this only applies to tiles of the same size. You can't combine x and -1 to get zero. ***A positive tile and a negative tile of the same shape create a zero pair.***

Representing Expressions

To represent an expression using algebra tiles, represent x^2, x, and constant using the appropriate number (and color) of tiles. An advantage to virtual or hand-drawn tiles is that you can't run out of pieces (sometimes an issue with physical sets). Can you determine what expression is represented in the image below? The answer is in the caption.

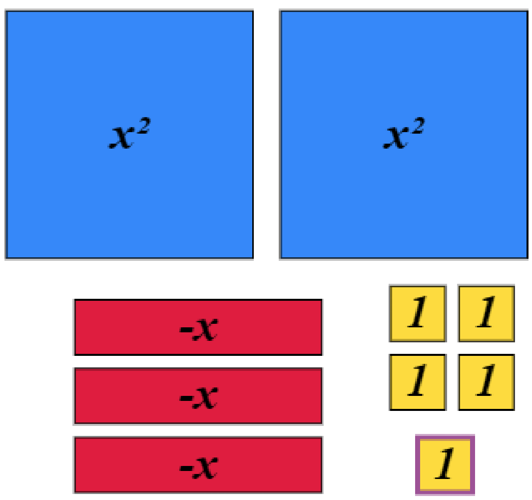

Figure 3: $2x^2$-3x+5

You can use algebra tiles to model solving equations. The key idea is that ***the equation is unchanged if the same thing is added to both sides of the equation.***

Consider **3x+1=7**. First represent the problem.

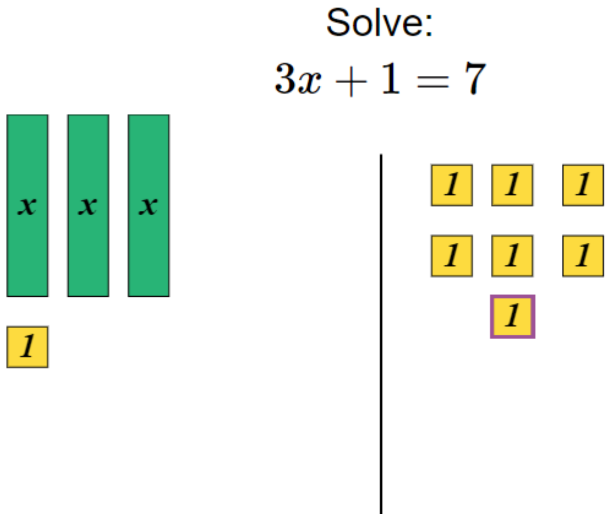

Figure 4: 3x+1=7 setup

Next, subtract 1 from both sides (add a -1):

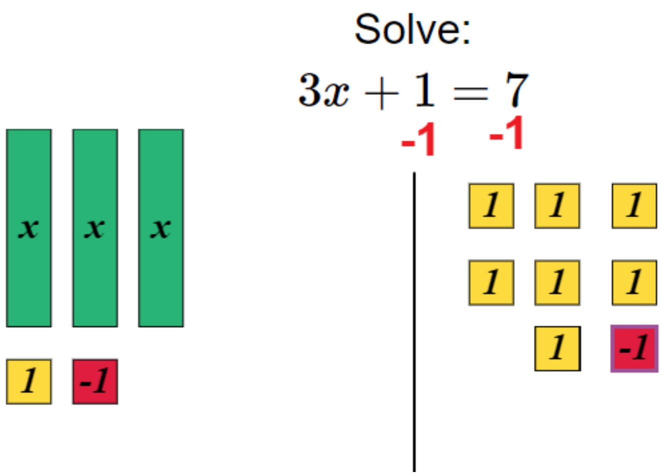

Figure 5: Subtract 1 from both sides

Note that there is a zero pair on both sides that cancels out, so now we have 3x = 6. Divide by 3 by shifting both sides into three groups. The answer is the number in one group, x = 2.

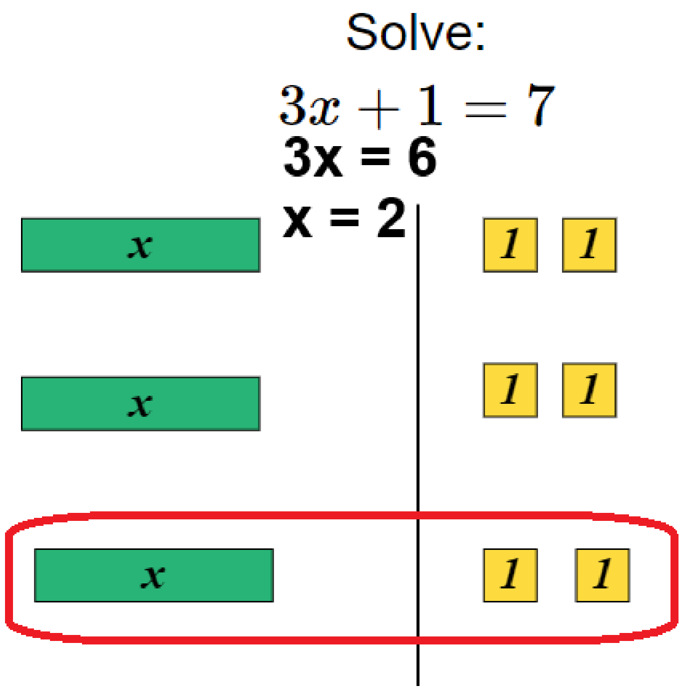

Figure 6: 3x = 6, so x = 2

Multiplying Polynomials

We can multiply polynomials in a fashion similar to the area model. Represent one polynomial vertically and the other horizontally. Consider **(2x + 3)(x + 1)**. First, represent the problem:

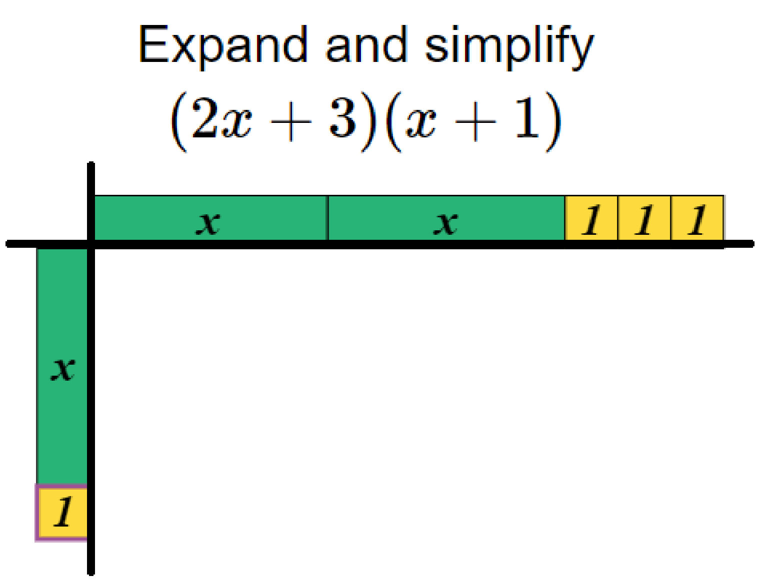

Figure 7: (2x+3)(x+1) setup

Now multiply the pieces. x times x is x^2, x times 1 is x, and 1 times 1 is one. Note that this fills in a box inside the grid, and that edges line up nicely, with no overlaps or gaps.

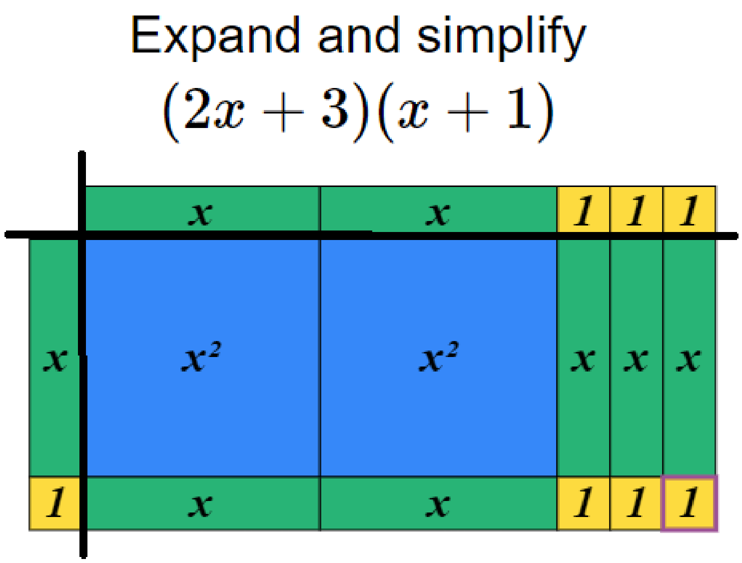

Figure 8:Expand out multiplication

$$(2x+3)(x+1)$$
$$2x^2+3x+2x+3$$
$$2x^2+5x+3$$

Note that you can connect the multiplication with the written work as well as the "FOIL" mnemonic.

The answer is the square inside the box. In this case, $2x^2+3x+2x+3=2x^2+5x+3$.

Note that this works with negative tiles, too; we just have to make sure to account for zero pairs.

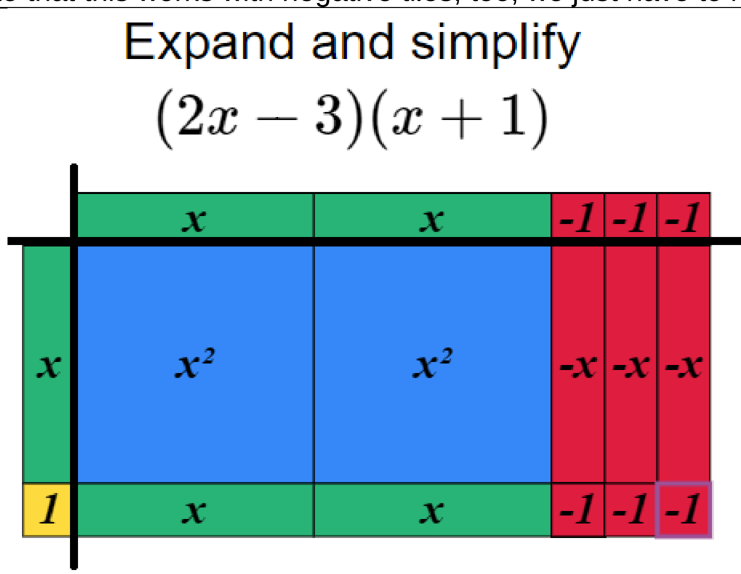

Figure 9: Expand multiplication with different signs

$$(2x-3)(x+1)$$
$$2x^2-3x+2x-3$$
$$2x^2-x-3$$

Note that you can connect the multiplication with the written work as well as the "FOIL" mnemonic.

Note how zero pairs cancel out. Also note that positive x's and negative x's don't touch each other. This will be important when we factor.

Factoring Polynomials

When we factor a polynomial, we break it down into simpler polynomials. This is often taught by guess and check methods and can be confusing for students. This is essentially the opposite of what we were doing when we multiplied polynomials. We want to fill our polynomial into the square inside the box, then back up to find the factors that give us that arrangement.

Consider **$2x^2+5x+3$**. First, represent the polynomial

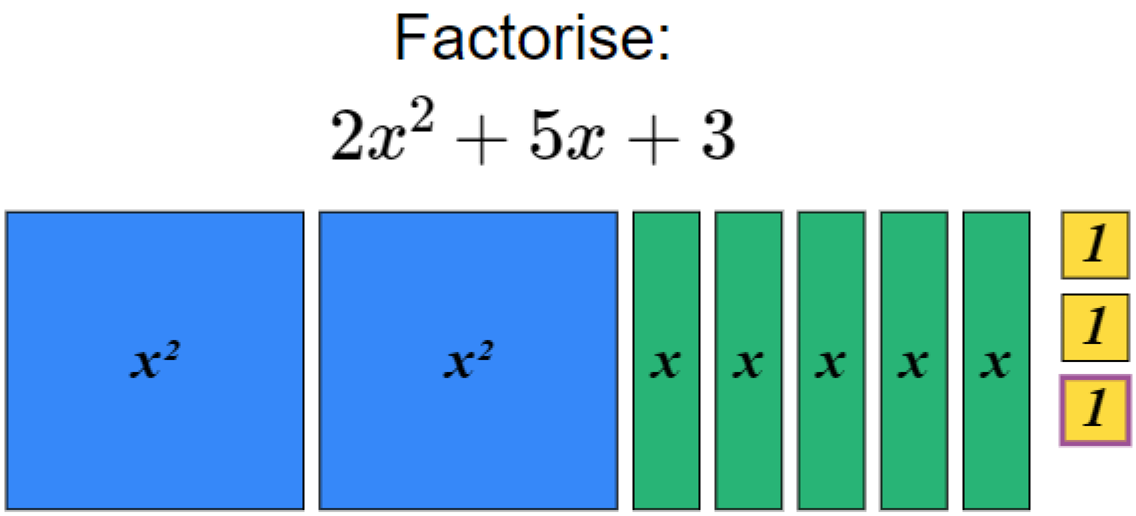

Figure 10: $2x^2+5x+3$

Now we want to arrange our tiles into a box. Note in the multiplication examples that the x^2's are all in the top left corner, and the 1's are in the bottom right. The x's are split into two sections. If you have a mix of positive and negative, there's one group of positive and one group of negative--they don't mix.

So start by putting the x^2's in the corner. Then consider how to split of 3. I can have 1x3 or 3x1. If you think about where you need x's to fill in the edges, you see one arrangement requires 7 x's and the other requires 5. We have 5 x's, so we'll go with the second arrangement.

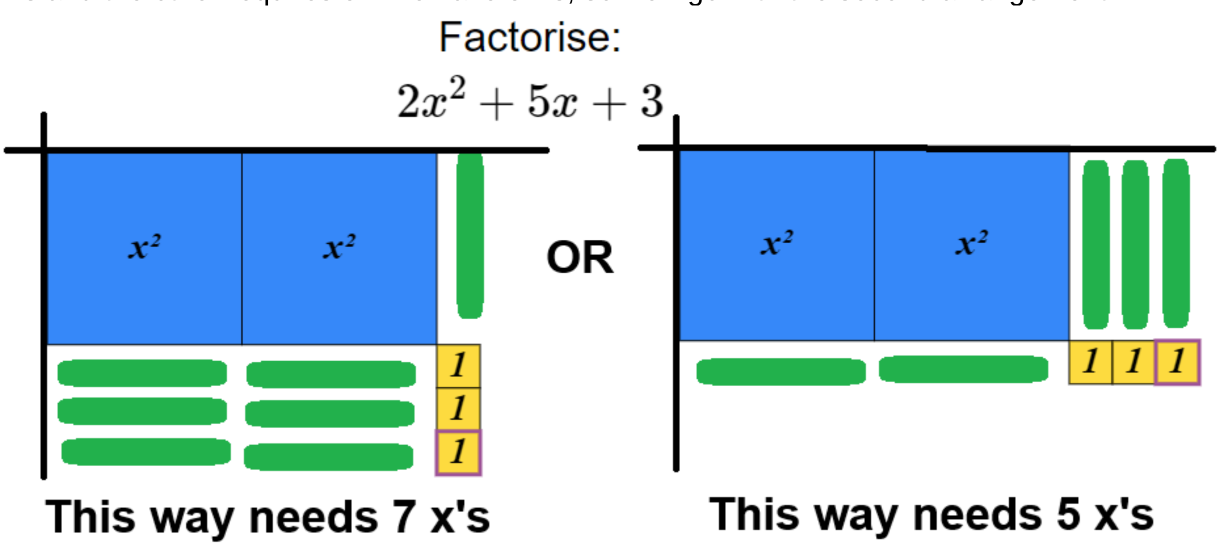

Figure 11: Two possible arrangements of 1's.

Note that if we don't have enough x's, we can add more by adding zero pairs. But then we have to worry about not mixing positives and negatives. We'll do an example like this next.

Now we fill in our x's, and backtrack to the factors. x^2 comes from x times x. x comes from 1 times x. 1 comes from 1 times 1. We can see that the factors are (2x+3)(x+1).

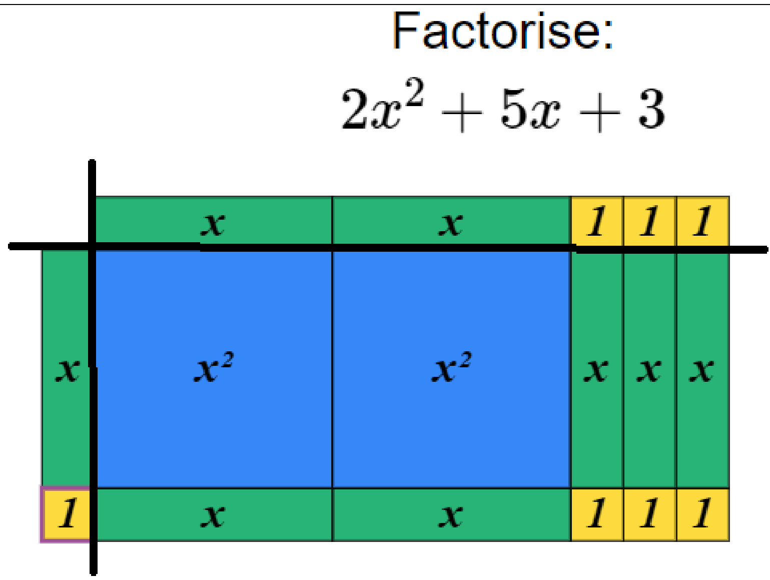

Figure 12: Finding factors

$$2x^2 + 5x + 3$$
$$= (2x + 3)(x + 1)$$
$$= (x + 1)(2x + 3)$$

Note that the factors could be listed in either order; as long as the pieces are the same, the solution is equivalent (2x3=3x2).

You can FOIL this back out to check your answer.

What if there aren't enough x's to start? We can add more with zero pairs.
Consider $\mathbf{x^2 + 2x - 8}$.

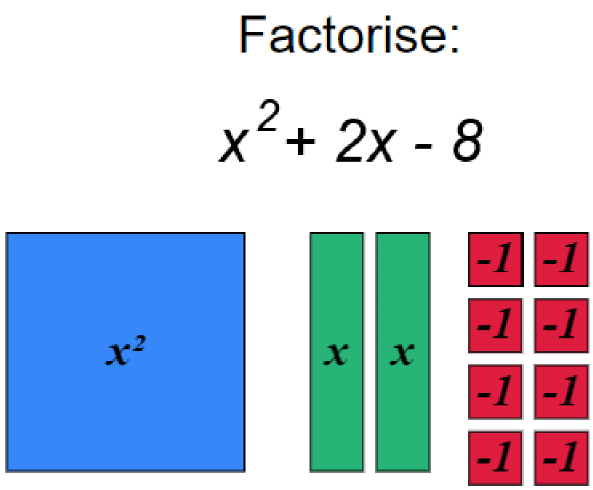

Figure 13: $x^2 + 2x -8$

We can arrange our 1's as 1x8 or 2x4. 1x8 will need 9 x's to fill in the box. 2x4 will need 6 x's to fill in the box.

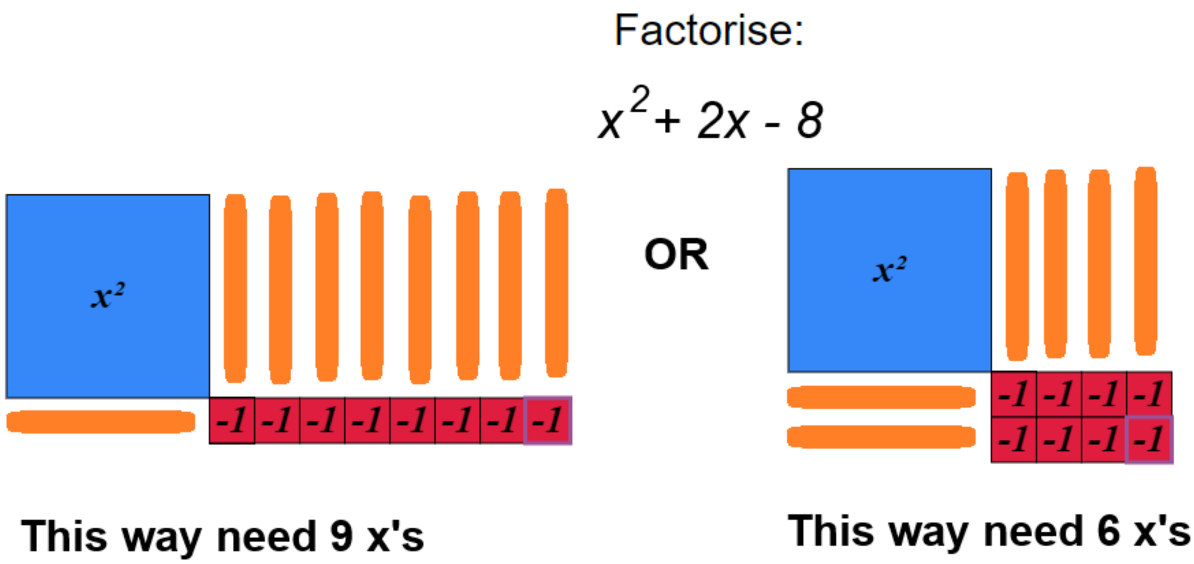

Figure 14: Two possible arrangements for factoring

We only have 2 x's. We can get more by adding zero pairs; that means we can add x's in multiples of 2. From 2 to 9 needs 7 more—there's no way to do that. From 2 to 6 needs 4 more—I can do that with 2 zero pairs. That means I have 4 positive x's and 2 negative x's. When I arrange my x's, I can't mix positive and negative, which means that my 2 negative x's will go on bottom and the 4 positives will go on the side. Note that I still have $x^2 + 2x - 8$.

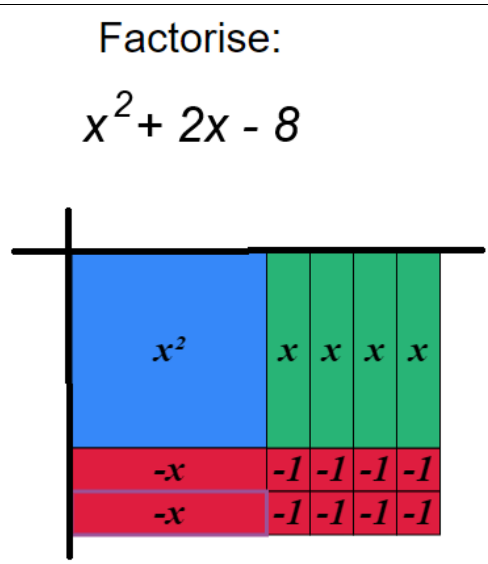

Figure 15: $x^2 + 2x - 8$ arranged for factoring

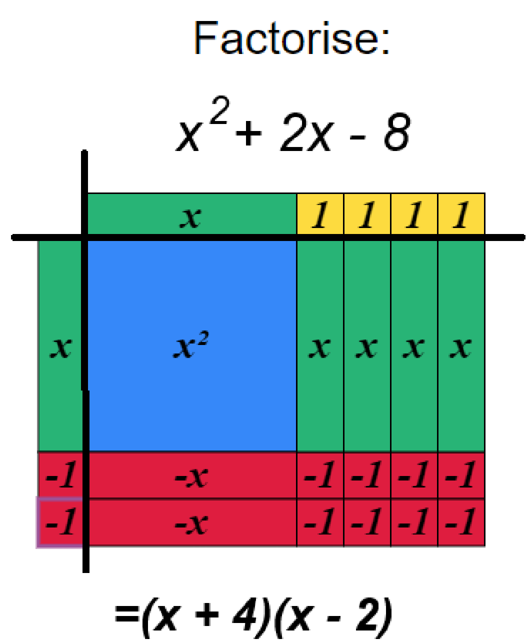

Figure 16: Factored as (x+4)(x-2)

Then you can backtrack to your factors. In this case, (x + 4) (x -2).

Remember that the point of this activity isn't to find the factors, then make it fit the model. The point is to use the model to help us find the factors. It's probably harder if already know how to model, because you're trying to fill in the concept under the algorithm. Keep in mind your students will learn the model first, then move on to doing problems without the tiles.

Dividing Polynomials

Dividing polynomials using algebra tiles if very similar to factoring—except you know one of the factors. Arrange the divisor into the box, with one factor known.

Consider (x^2 + 7x + 6) ÷ (x + 1). We represent out polynomial with the tiles, and fill in a box underneath the divisor. The same rules apply—nice box, clean lines, no overlaps, don't mix positive and negative x's. If you need more x's you can add zero pairs. Then, backtrack to find the other factor. In this case, it's x+6.

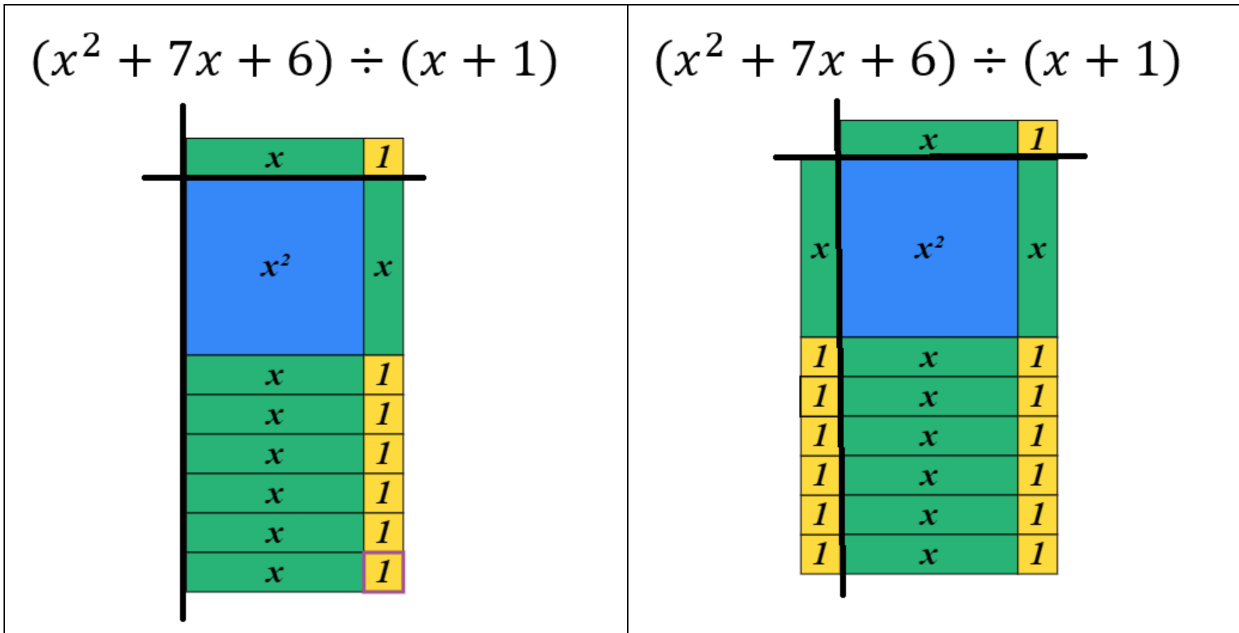

Figure 17: Dividing polynomials